INTERNATIONAL INSTITUTE OF TECHNOLOGY & MANAGEMENT, MURTHAL, SONEPAT

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UNIT – 1

INTRODUCTION TO OPEN CHANNEL FLOW

1.1 What is Open Channel Flow?

Flow in open channels is defined as the flow of a liquid with a free surface. A free surface is a surface having constant pressure such as atmospheric pressure. Thus, a liquid flowing at atmospheric pressure through a passage is known as flow in open channels. In most of cases, the liquid is taken as water. Hence flow of water through a passage under atmospheric pressure is called flow in open channels. The flow of water through pipes at atmospheric pressure or when the level of water in the pipe is below the top of the pipe, is also classified as open channel flow.

In case of open channel flow, as the pressure is atmospheric, the flow takes place under the force of gravity which means the flow takes place due to the slope of the bed of the channel only. The hydraulic gradient line coincides with the free surface of water.

Simply stated, Open channel flow is a flow of liquid in a conduit with free space. Open channel flow is particularly applied to understand the flow of a liquid in artificial (flumes, spillways, canals, weirs, drainage ditch, culverts) and natural (streams, rivers, flood plains). The two kinds of flow are similar in many ways but differ in one important respect. Open-channel flow must have a *free surface*, whereas pipe flow has none.

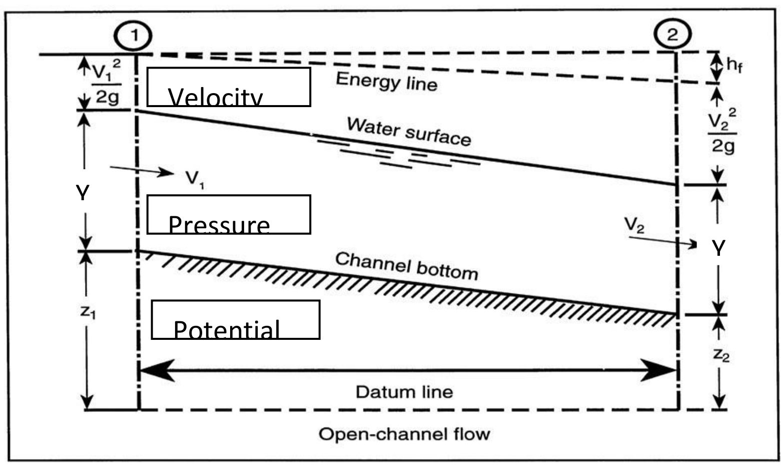


Figure 1: Schematic Presentation of Open Channel

1.2 Classification of Open Channel Flows:

A channel in which the cross-sectional shape and size and also the bottom slopes are constant is termed as a prismatic channel. Most of the man-made (artificial) channels are prismatic channels over long stretches. The rectangle, trapezoid, triangle and circle are some of the commonly used shapes in made channels. All natural channels generally have varying cross-sections and consequently are non-prismatic.

a) <u>Steady and Unsteady Open Channel Flow:</u> If the flow depth or discharge at a cross-section of an open channel flow is not changing with time, then the flow is steady flow, otherwise it is called as unsteady flow. Flood flows in rivers and rapidly varying surges in canals are some examples of unsteady flows. Unsteady flows are considerably more difficult to analyze than steady flows.

b) <u>Uniform and Non-Uniform Open Channel Flow</u>: If the flow depth along the channel is not changing at every cross-section for a taken time, then the flow is uniform flow. If the flow depth changes at every cross-section along the flow direction for a taken time, then it is non-uniform flow.

A prismatic channel carrying a certain discharge with a constant velocity is an example of uniform flow.

c) Uniform Steady Flow: The flow depth does not change with time at every cross section and at the same time is constant along the flow direction. The depth of flow will be constant along the channel length and hence the free surface will be parallel to the bed.

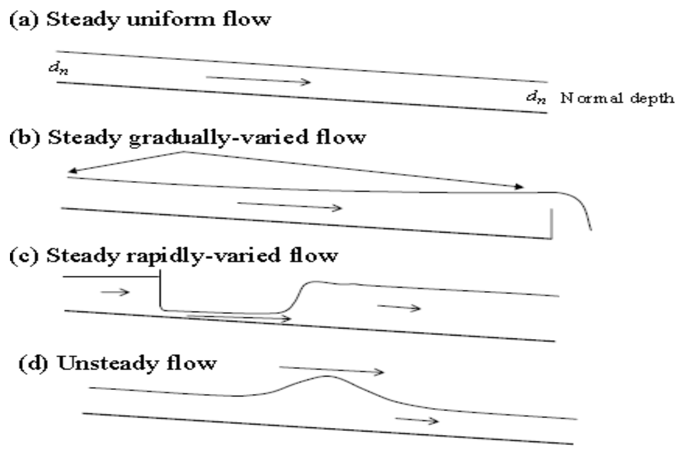


Figure 2: Schematic representation of different types of open channel flow

d) Non-Uniform Steady Flows: The water depth changes along the channel cross sections but does not change with time at every cross section with time. A typical example of this kind of flow is the backwater water surface profile at the upstream of a dam.

e) Rapidly Varied Flow (R.V.F.): Rapidly varied flow is defined as that flow in which depth of flow changes abruptly over a small length of the channel. When there is any obstruction in the path of flow of water, the level of water rises above the obstruction and then falls and again rises over a small length of channel. Thus, the depth of flow changes rapidly over a short length of the channel. For this short length of the channel the flow is called rapidly varied flow (R.V.F.).

f) Gradually Varied Flow (G.V.F.): If the depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be gradually varied flow and is denoted by G.V.F.

g) Laminar Flow and Turbulent Flow: The flow in open channel is said to be laminar if the Reynold number (Re) is less than 500 or 600. Reynold number in case of open channels is defined as: R_e= pVR

where V = Mean velocity of flow of water, R = Hydraulic radius or Hydraulic mean depth

=Cross-section area of flow normal to the direction of flow/ Wetted perimeter

p and μ = Density and viscosity of water.

If the Reynold number is more than 2000, the flow is said to be turbulent in open channel flow. If R_e lies between 500 to 2000, the flow is considered to be in transition state.

1.3 Difference between Open Channel & Pipe Flow

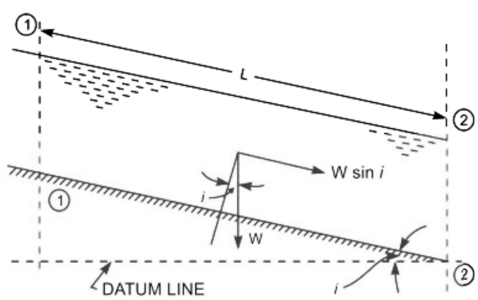
Despite the similarity between the two kinds of flow, it is much more difficult to solve problems of flow in open channels than in pipes. Flow conditions in open channels are complicated by the position of the free surface which will change with time and space. And also by the fact that depth of flow, the discharge, and the slopes of the channel bottom and of the free surface are all inter-dependent.

Physical conditions in open-channels vary much more than in pipes - the cross-section of pipes is usually round - but for open channel it can be any shape. Treatment of roughness also poses a greater problem in open channels than in pipes. Although there may be a great range of roughness in a pipe from polished metal to highly corroded iron, open channels may be of polished metal to natural channels with long grass and roughness that may also depend on depth of flow. And also, Open channel flows are found in large and small scale. Open channel flow is driven by gravity rather than by pressure work as in pipes.

	Pipe flow	**Open Channel flow**
Flow driven by	Pressure work	Gravity (potential energy)
Flow cross section	Known, fixed	Unknown in advance because the flow depth is unknown
Characteristics parameters	flow velocity deduced from continuity	Flow depth deduced simultaneously from solving both continuity and momentum equations
Specific boundary		Atmospheric pressure at conditions the free surface

- **1.4 Discharge through Open Channel by Chezy's Constant**

Consider uniform flow of water in a channel as shown in Fig. 16.2. As the flow is uniform, it means the velocity, depth of flow and area of flow will be constant for a given length of the channel. Consider sections 1-1 and 2-2.



Let L = Length of channel, A= Area of flow of water, i = Slope of the bed, V = Mean velocity of flow of water, P = Wetted perimeter of the cross-section, f = Frictional resistance per unit velocity per unit area.

- The weight of water between sections 1-1 and 2-2.
  W = Specific weight of water x volume of water =wxAxL
- Component of W along direction of flow = W x sin i = wAL sin i
- Frictional resistance against motion of water= f x surface area x $(\text{velocity})^n$
- The value of n is found experimentally equal to 2 and surface area = P x L
- Frictional resistance against motion = f x P x L x V^2

The forces acting on the water between sections 1-1 and 2-2 are:

1. Component of weight of water along the direction of flow,
2. Friction resistance against flow of water,
3. Pressure force at section 1-1,
4. Pressure force at section 2-2.

As the depths of water at the sections 1-1 and 2-2 are the same, the pressure forces on these two sections are same and acting in the opposite direction. Hence they cancel each other. In case of uniform flow, the velocity of flow is constant for the given length of the channel. Hence there is no acceleration acting on the water. Hence the resultant force acting in the direction of flow must be zero.

∴ Resolving all forces in the direction of flow, we get

$$wAL \sin i - f \times P \times L \times V^2 = 0$$

or $$wAL \sin i = f \times P \times L \times V^2$$

$$V^2 = \frac{wAL \sin i}{f \times P \times L} = \frac{w}{f} \times \frac{A}{P} \times \sin i$$

or $$V = \sqrt{\frac{w}{f}} \times \sqrt{\frac{A}{P} \times \sin i}$$

But $$\frac{A}{P} = m$$

= hydraulic mean depth or hydraulic radius,

$$\sqrt{\frac{w}{f}} = C = \text{Chezy's constant}$$

Substituting the values of $\frac{A}{P}$ and $\sqrt{\frac{w}{f}}$ in equation (*iii*), $V = C\sqrt{m \sin i}$

For small values of i, $\sin i \simeq \tan i \approx i$ ∴ $V = C\sqrt{mi}$

∴ Discharge, $Q = \text{Area} \times \text{Velocity} = A \times V$

$= A \times C\sqrt{mi}$

Numerical related to Chezy's constant:

Q1. Find the velocity of flow and rate of flow of water through a rectangular channel of 6 m wide and 3 m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take Chery 's constant C = 55.

Width of rectangular channel, $b = 6$ m

Depth of channel, $d = 3$ m

∴ Area, $A = 6 \times 3 = 18 \text{ m}^2$

Bed slope, $i = 1 \text{ in } 2000 = \frac{1}{2000}$

Chezy's constant, $C = 55$

Perimeter, $P = b + 2d = 6 + 2 \times 3 = 12$ m

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{18}{12} = 1.5$ m

Velocity of flow is given by equation (16.4) as,

$$V = C\sqrt{mi} = 55\sqrt{1.5 \times \frac{1}{2000}} = \mathbf{1.506\ m/s.\ Ans.}$$

Rate of flow, $Q = V \times \text{Area} = V \times A = 1.506 \times 18 =$ **27.108 $\mathbf{m^3/s}$. Ans.**

Explanation: In the question width & depth of the channel was provided, using which Area is determined first. Later perimeter is calculated. After which we have area & perimeter, using which we can calculate the hydraulic mean depth of the channel. To calculate the velocity, we have slope and m, along with the C provided in question.

Rate of flow is nothing but Discharge in the channel, which is Velocity * Area of the channel.

Q2. Find the slope of the bed of a rectangular channel of width 5 m when depth of water is 2 m and rate of flow is given as 20 m^3/s. Take Chery's constant, C = 50.

Solution: Width = 5m Depth = 2m Q = 20 m^3/s

Chezy's constant $C = 50$

Let the bed slope $= i$

Using equation (16.5), we have $Q = AC\sqrt{mi}$

where $A = \text{Area} = b \times d = 5 \times 2 = 10 \text{ m}^2$

$$m = \frac{A}{P} = \frac{10}{b+2d} = \frac{10}{5+2\times 2} = \frac{10}{5+4} = \frac{10}{9} \text{ m}$$

$$\therefore \quad 20.0 = 10 \times 50 \times \sqrt{\frac{10}{9} \times i} \text{ or } \sqrt{\frac{10}{9}\, i} = \frac{20.0}{500} = \frac{2}{50}$$

Squaring both sides, we have $\frac{10}{9} i = \frac{4}{2500}$

$$\therefore \quad i = \frac{4}{2500} \times \frac{9}{10} = \frac{36}{25000} = \frac{1}{\frac{25000}{36}} = \frac{1}{\mathbf{694.44}}. \textbf{ Ans.}$$

∴ Bed slope is 1 in 694.44.

Geometric Properties Necessary for Analysis:

HYDRAULICS: ELEMENTS OF CHANNEL SECTIONS

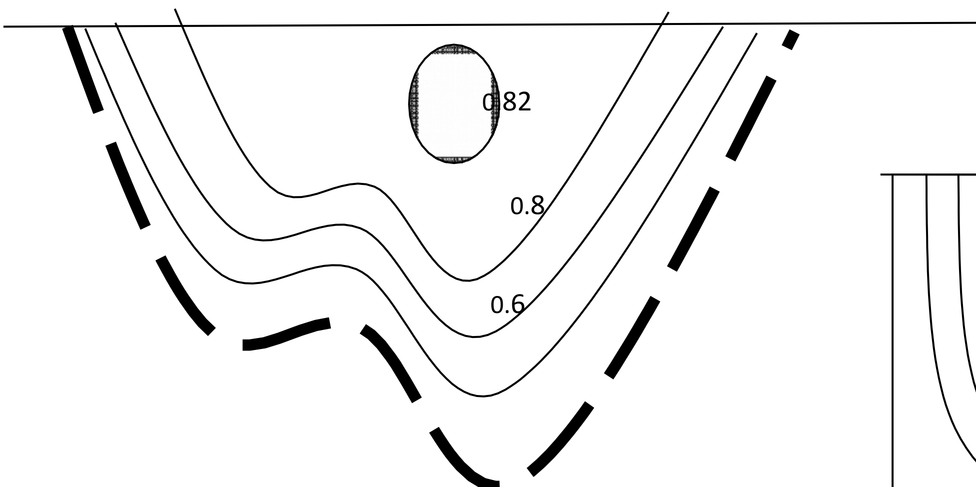
Section	Area a	Wetted Perimeter p	Hydraulic Radius r	Top Width T
Trapezoid	$bd+zd^2$	$b+2d\sqrt{z^2+1}$	$\frac{bd+zd^2}{b+2d\sqrt{z^2+1}}$	$b+2zd$
Rectangle	bd	$b+2d$	$\frac{bd}{b+2d}$	b
Triangle	zd^2	$2d\sqrt{z^2+1}$	$\frac{zd}{2\sqrt{z^2+1}}$	$2zd$
Parabola	$\frac{2}{3}dT$	$T+\frac{8d^2}{3T}$ [1]	$\frac{2dT^2}{3T^2+8d^2}$ [1]	$\frac{3a}{2d}$
Circle - < ½ full [2]	$\frac{D^2}{8}\left(\frac{\pi\theta}{180}-\sin\theta\right)$	$\frac{\pi D\theta}{360}$	$\frac{45D}{\pi\theta}\left(\frac{\pi\theta}{180}-\sin\theta\right)$	$D\sin\frac{\theta}{2}$ or $2\sqrt{d(D-d)}$
Circle - > ½ full [3]	$\frac{D^2}{8}\left(2\pi-\frac{\pi\theta}{180}+\sin\theta\right)$	$\frac{\pi D(360-\theta)}{360}$	$\frac{45D}{\pi(360-\theta)}\left(2\pi-\frac{\pi\theta}{180}+\sin\theta\right)$	$D\sin\frac{\theta}{2}$ or $2\sqrt{d(D-d)}$

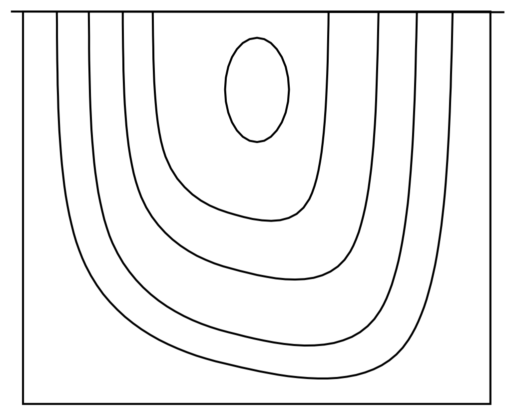
[1] Satisfactory approximation for the interval $0 < \frac{d}{T} \leq 0.25$. When $d/T > 0.25$, use $p = \frac{1}{2}\sqrt{16d^2+T^2} + \frac{T^2}{8d}\sinh^{-1}\frac{4d}{T}$

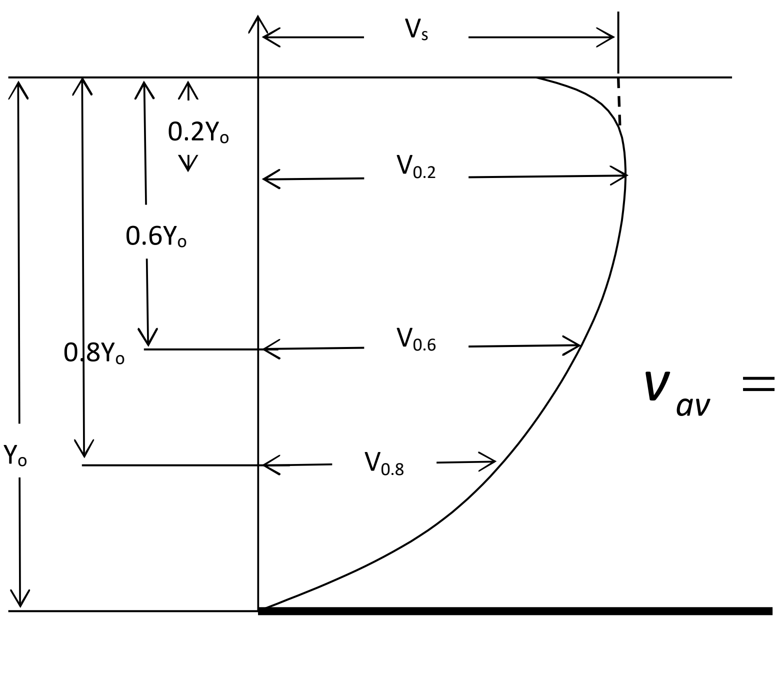
[2] $\theta = 4\sin^{-1}\sqrt{d/D}$
[3] $\theta = 4\cos^{-1}\sqrt{d/D}$
} Insert θ in degrees in above equations

1.5 Velocity Distribution in Open Channel:

Naturally three types of velocity are occurred in open channel flow, namely longitudinal the one along the flow direction, lateral at the bedside of the channel and normal perpendicular to the flow direction. However, the two velocities (the lateral and normal) are insignificance as compared to the longitudinal velocity. So we consider the longitudinal velocity in the analysis of velocity distribution. The longitudinal velocity commonly represent with v and its value is minimum at the bedside and gradually increase with the distance from the boundary and its maximum occur at a certain distance below the free surface around the center of the channel.







$$v_{av} = v_{0.6} = \frac{v_{0.2} + v_{0.8}}{2}$$

References:

- Book – “Flow in Open Channel” By – K. Subramanya
- Book – “Fluid Mechanics” By - R.K Bansal

UNIT – 3

Design of Channel Section

3.1 Most Efficient Channel Sections

A section of a channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction of a channel depends upon the excavation and the lining. To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum. This condition is utilized for determining the dimensions of an economical sections of different form of channels.

Most economical section is also called the best section or most efficient section as the discharge, assing through a most economical section of channel for a given cross-sectional area (A), slope of the bed (i) and a resistance co-efficient, is maximum. But the discharge, Q is given by equation as

$$Q = AC\sqrt{mi} = AC\sqrt{\frac{A \times i}{P}} \qquad \left(\because \ m = \frac{A}{P}\right)$$

For a given A, i and resistance co-efficient C, the above equation is written as

$$Q = K\frac{1}{\sqrt{P}}, \quad \text{where } K = AC\sqrt{Ai} = \text{constant}$$

Hence the discharge, Q will be maximum, when the wetted perimeter P is minimum. This condition will be used for determining the best section of a channel i.e., best dimensions of a channel for a given area.

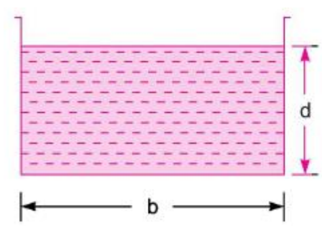
The conditions to be most economical for the following shapes of the channels will be considered :

1. Rectangular Channel,
2. Trapezoidal Channel, and
3. Circular Channel.

3.1.1 Most Efficient Rectangular Channel:

The condition for most economical section, is that for a given area, the perimeter should be minimum.

Consider a rectangular channel as shown in Fig.



Let, b = Width of Channel
d = Depth of the flow then Area, A = b * d

& Wetted Perimeter P = b + 2d

From above we get b = A / d,

Substituting this value in case of wetted perimeter, we get P = b + 2d = A/d + 2d

Now, for most efficient channel section, P should be minimum for a given area.

or $$\frac{dP}{d(d)} = 0$$

Differentiating the equation *(iii)* with respect to d and equating the same to zero, we get

$$\frac{d}{d(d)}\left[\frac{A}{d} + 2d\right] = 0 \quad \text{or} \quad -\frac{A}{d^2} + 2 = 0 \quad \text{or} \quad A = 2d^2$$

But, A = b * d which implies > b * d = $2d^2$ or **b = 2d** **(i)**

Now hydraulic mean depth, $m = \frac{A}{P} = \frac{b \times d}{b + 2d}$ $(\because \quad A = bd, P = b + 2d)$

= 2d × d / 2d + 2d = **$2d^2$/ 4d** **(ii)**

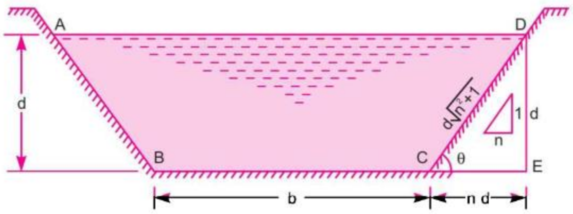
From eq^n (i) & (ii), it is clear that the rectangular channel will be most economical when:

- Either b = 2d which means width is two times the depth of flow.
- Or m = d/2 means hydraulic depth is half of depth of flow.

3.1.2 Most Efficient Trapezoidal Channel Section:

The trapezoidal section of a channel will be most economical when its wetted perimeter is minimum.

Consider a trapezoidal channel section as shown in fig. below:



Let, b = width of channel Bottom
d = Depth of the flow
ø = angle made by the sides with horizontal

(*i*) The side slope is given as 1 vertical to *n* horizontal.

$\therefore$ Area of flow, $A = \frac{(BC + AD)}{2} \times d = \frac{b + (b + 2nd)}{2} \times d$ $(\because \quad AD = b + 2nd)$

$$= \frac{2b + 2nd}{2} \times d = (b + nd) \times d \quad ...(i)$$

$$\therefore \quad \frac{A}{d} = b + nd$$

$$\therefore \quad b = \frac{A}{d} - nd \quad ...(ii)$$

Now wetted perimeter, $P = AB + BC + CD = BC + 2CD$ $(\because \quad AB = CD)$

$$= b + 2\sqrt{CE^2 + DE^2} = b + 2\sqrt{n^2d^2 + d^2} = b + 2d\sqrt{n^2 + 1} \quad ...(iia)$$

Substituting the value of *b* from equation (*ii*), we get

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \quad ...(iii)$$

For most economical section, *P* should be minimum or $\frac{dP}{d(d)} = 0$

$\therefore$ Differentiating equation (*iii*) with respect to *d* and equating it equal to zero, we get

$$\frac{d}{d(d)}\left[\frac{A}{d} - nd + 2d\sqrt{n^2 + 1}\right] = 0$$

or $$-\frac{A}{d^2} - n + 2\sqrt{n^2 + 1} = 0 \quad (\because \quad n \text{ is constant})$$

or $$\frac{A}{d^2} + n = 2\sqrt{n^2 + 1}$$

Substituting the value of *A* from equation (*i*) in the above equation,

$$\frac{(b + nd)d}{d^2} + n = 2\sqrt{n^2 + 1} \quad \text{or} \quad \frac{b + nd}{d} + n = 2\sqrt{n^2 + 1}$$

or $$\frac{b+nd+nd}{d} = \frac{b+2nd}{d} = 2\sqrt{n^2+1} \quad \text{or} \quad \frac{b+2nd}{2} = d\sqrt{n^2+1}$$

But, $$\frac{b+2nd}{2} = \text{Half of top width}$$

$$d\sqrt{n^2+1} = CD = \text{one of the sloping side}$$

From above equation is the required condition for a trapezoidal section to be most economical, which can be expressed as half of the top width must be equal to one of the sloping sides of the channel.

(ii) Hydraulic Mean Depth

Hydraulic mean depth, $m = \frac{A}{P}$

Value of A from (i), $A = (b + nd) \times d$

Value of P from (iia), $P = b + 2d\sqrt{n^2+1} = b + (b + 2nd)$ Also, $b + 2nd = 2d\sqrt{n^2+1}$)

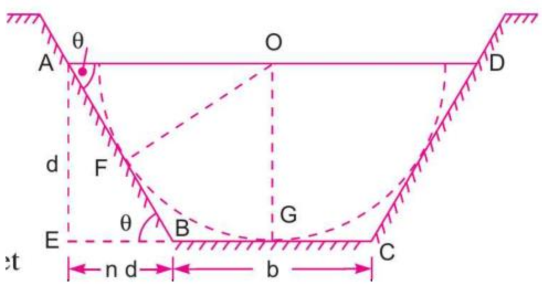
$$= 2b + 2nd = 2(b + nd)$$

$$\therefore \text{ Hydraulic mean depth, } m = \frac{A}{P} = \frac{(b+nd)d}{2(b+nd)} = \frac{d}{2}$$

Hence for a trapezoidal section to be most economical hydraulic mean depth must be equal to half the depth of flow,

(iii) The three sides of the trapezoidal section of most economical section are tangential to the semi-circle described on the water line. This is proved as:

Let Fig. below shows the trapezoidal channel of most economical section.



Let ø = angle made by the sloping side with horizontal, and

O = the centre of the top width, AD.

Draw OF perpendicular to the sloping side AB.

Triangle OAF is a right-angled triangle and angle OAF = ø

$$\therefore \quad \sin\theta = \frac{OF}{OA} \qquad \therefore \quad OF = AO \sin\theta$$

In ΔAEB, $$\sin\theta = \frac{AE}{AB} = \frac{d}{\sqrt{d^2+n^2d^2}}$$

$$= \frac{d}{d\sqrt{1+n^2}} = \frac{1}{\sqrt{1+n^2}}$$

Substituting $\sin\theta = \frac{1}{\sqrt{1+n^2}}$ in equation

$$OF = AO \times \frac{1}{\sqrt{1+n^2}}$$

But AO = half of top width

$$= \frac{b+2nd}{2} = d\sqrt{n^2+1}$$

Substituting this value of AO in above equation, we get

$$OF = \frac{d\sqrt{n^2+1}}{\sqrt{n^2+1}} = d \text{ depth of flow}$$

Thus, if a semi-circle is drawn with O as centre and radius equal to the depth of flow d, the three sides of most economical trapezoidal section will be tangential to the semi-circle.

Hence the conditions for the most economical trapezoidal section are:

- $\frac{b+2nd}{2} = d\sqrt{n^2+1}$
- m = d/ 2
- A semi-circle drawn from O with radius equal to depth of flow will touch the three sides of the channel.

3.1.3 Best Side Slope for Most Economical Trapezoidal Section

Area of trapezoidal section, $A = (b + nd)d$ ...(*i*)

where b = width of trapezoidal channel, d = depth of flow, and

n = slope of the side of the channel

From equation (*i*), $b = \frac{A}{d} - nd$ ...(*ii*)

Perimeter (wetted) of channel, $P = b + 2d\sqrt{n^2+1}$

Substituting the value of b from equation (*ii*), perimeter becomes

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2+1} \quad ...(iii)$$

For the most economical trapezoidal section depth of flow d, and area A, is the only variable. Best side slope will be when section is most economical or in other words, P is minimum. For P minimum we have, dP/dn = 0

Hence differentiating equation (*iii*) with respect to n,

$$\frac{d}{dn}\left[\frac{A}{d} - nd + 2d\sqrt{n^2+1}\right] = 0$$

or $-d + 2d \times \frac{1}{2} \times (n^2+1)^{1/2-1} \times 2n = 0$ or $-d + 2nd \times \frac{1}{\sqrt{n^2+1}} = 0$

Cancelling d and re-arranging, we get $2n = \sqrt{n^2+1}$

Squaring to both sides,

$$4n^2 = n^2 + 1 \text{ or } 3n^2 = 1 \text{ or } n = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

If the sloping side makes and Angle ø , with the horizontal then we have,

Tan ø = 1/n = 1/ 1√3 = tan 60°

Ø = 60°

Hence best side slope is at 60 to the horizontal or the value of n for the best side slope is given by eq^{n}

$$4n^2 = n^2 + 1 \text{ or } 3n^2 = 1 \text{ or } n = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

Half of top width = length of one sloping side

or $$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

Substitute the value of n from above equation:

$$\frac{b + 2 \times \frac{1}{\sqrt{3}} \times d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} = \frac{2d}{\sqrt{3}} \quad \text{or} \quad \frac{\sqrt{3}b + 2d}{2 \times \sqrt{3}} = \frac{2d}{\sqrt{3}}$$

or $$\sqrt{3}b + 2d = 2 \times \sqrt{3} \times \frac{2d}{\sqrt{3}} = 4d$$

∴ $$b = \frac{4d - 2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}} \qquad ...(iv)$$

Now, wetted perimeter, $$P = b + 2d\sqrt{n^2 + 1}$$

$$= \frac{2d}{\sqrt{3}} + 2d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} \qquad \left(\because\ b = \frac{2d}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2d}{\sqrt{3}} + 2d \times \frac{2}{\sqrt{3}} = \frac{2d}{\sqrt{3}} + \frac{4d}{\sqrt{3}}$$

or $$P = \frac{6d}{\sqrt{3}} = 3 \times \frac{2d}{\sqrt{3}} = 3 \times b \qquad \left(\because\ \text{From } (iv), \frac{2d}{\sqrt{3}} = b\right)$$

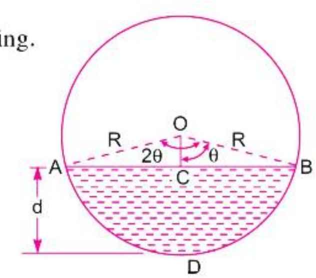
For a slope of 60°, the length of sloping side is equal to the width of the trapezoidal section.

- **Flow through Circular Channel**

The flow of a liquid through a circular pipe, when the level of liquid in the pipe is below the top of the pipe is classified as an Open Channel flow. The rate of flow through circular channel is determined from the depth of flow and angle subtended by the liquid surface at the centre of the circular channel.

Fig. below shows a circular channel through which water is flowing.

ing.



Let d = depth of water,
2ø = angle subtended by water surface AB at the centre in radians
R = radius of the channel

Then the wetted perimeter and wetted area is determined as:

Wetted perimeter,

$$P = \frac{2\pi R}{2\pi} \times 2\theta = 2R\theta$$

Wetted area, A = Area $ADBA$

$$= \text{Area of sector } OADBO - \text{Area of } \Delta ABO$$

$$= \frac{\pi R^2}{2\pi} \times 2\theta - \frac{AB \times CO}{2} = R^2\theta - \frac{2BC \times CO}{2} \qquad (\because \quad AB = 2BC)$$

$$= R^2\theta - \frac{2 \times R\sin\theta \times R\cos\theta}{2} \qquad (\because \; BC = R\sin\theta,\; CO = R\cos\theta)$$

$$= R^2\theta - \frac{R^2 \times 2\sin\theta\cos\theta}{2} = R^2\theta - \frac{R^2\sin 2\theta}{2} \qquad (\because \; 2\sin\theta\cos\theta = \sin 2\theta)$$

$$= R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

Then hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta} = \dfrac{R}{2\theta}\left(\theta - \dfrac{\sin 2\theta}{2}\right)$

And discharge, Q is given by, $Q = AC\sqrt{mi}$.

- **Most Economical Circular Section**

As discussed above that for a most economical section the discharge for a constant cross-sectional area, slope of bed and resistance co-efficient, is maximum. But in case of circular channels, the area of flow cannot be maintained constant. With the change of depth of flow in a circular channel of any radius, the wetted area and wetted perimeter changes. Thus, in case of circular channels, for most economical section, two separate conditions are obtained. They are:

i. Conditions for maximum velocity, and
ii. Condition for maximum discharge.

1. Condition for Maximum Velocity for Circular Section:

Fig. shows a circular channel through which water is flowing.

Let, d = depth of water,
2ø = Angle subtended at the corner by water surface,
R = radius of channel, and
i = slope of the bed

The velocity of flow according to Chezy's formula is given as

$$V = C\sqrt{mi} = C\sqrt{\frac{A}{P}\, i} \quad \left(\because \quad m = \frac{A}{P}\right)$$

The Velocity of flow through a circular channel is maximum when the hydraulic mean depth m or A/P is maximum for a given value of C and i. In case of circular pipe, the variable is ø only.

Hence for maximum value A/P we have the condition,

$\dfrac{d\left(\dfrac{A}{P}\right)}{d\theta} = 0$ Where A & P both are function of ø.

The value of wetted area, A is given by equation as $A = R^2\left(\theta - \dfrac{\sin 2\theta}{2}\right)$

The value of wetted perimeter, P is given by eq^n as:

P = 2Rø

Differentiating eq^n with respect to ø, we have:

$$\frac{P\dfrac{dA}{d\theta} - A\dfrac{dP}{d\theta}}{P^2} = 0 \quad \text{or} \quad P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$$

From equation (*ii*), $\dfrac{dA}{d\theta} = R^2\left(1 - \dfrac{\cos 2\theta}{2} \times 2\right) = R^2(1 - \cos 2\theta)$

From equation (*iii*), $\dfrac{dP}{d\theta} = 2R$

Substituting the values of A, $P\dfrac{dA}{d\theta}$ and $\dfrac{dP}{d\theta}$ in equation (*iv*),

$$2R\theta\left[R^2(1 - \cos 2\theta)\right] - R^2\left(\theta - \frac{\sin 2\theta}{2}\right)(2R) = 0$$

or $$2R^3\theta(1 - \cos 2\theta) - 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

or $$\theta(1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

$$\theta - \theta\cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

or $$\theta\cos 2\theta = \frac{\sin 2\theta}{2} \quad \text{or} \quad \frac{\sin 2\theta}{\cos 2\theta} = 2\theta$$

∴ $$\tan 2\theta = 2\theta$$

The solution of this equation by hit and trial, gives

$$2\theta = 257^\circ\ 30' \qquad \text{(approximately)}$$

or $$\theta = 128^\circ\ 45'$$

The depth of flow for maximum velocity from Fig. 16.24, is

$$d = OD - OC = R - R\cos\theta$$
$$= R[1 - \cos\theta] = R[1 - \cos 128^\circ\ 45'] = R[1 - \cos(180^\circ - 51^\circ\ 15')]$$
$$= R[1 - (-\cos 51^\circ\ 15')] = R[1 + \cos 51^\circ\ 15']$$
$$= R[1 + 0.62] = 1.62\ R = 1.62 \times \frac{D}{2} = 0.81\ D$$

Where D = diameter of the circular channel.

Thus, for maximum velocity of flow, the depth of water in the circular channel should be equal to 0.81 times the diameter of the channel.

Hydraulic mean depth for maximum velocity is:

$$m = \frac{A}{P} = \frac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta} = \frac{R}{2\theta}\left[\theta - \frac{\sin 2\theta}{2}\right]$$

where $\theta = 128° \; 45' = 128.75°$

$$= 128.75 \times \frac{\pi}{180} = 2.247 \text{ radians}$$

$$\therefore \quad m = \frac{R}{2 \times 2.247}\left[2.247 - \frac{\sin 257° \, 30'}{2}\right] = \frac{R}{4.494}\left[2.247 - \frac{\sin(180° + 87.5°)}{2}\right]$$

$$= \frac{R}{4.494}\left[2.247 + \frac{\sin 87.5°}{2}\right] = 0.611 \; R$$

$$= 0.611 \times \frac{D}{2} = 0.3055 \; D = 0.3 \; D$$

Thus, for maximum velocity, the hydraulic mean depth is equal to 0.3 times the diameter of circular channel.

- Condition for Maximum Discharge for Circular Section

Discharge through a channel is given by:

$$Q = AC\sqrt{mi} = AC\sqrt{\frac{A}{P}i}$$

$$= C\sqrt{\frac{A^3}{P}i}$$

The discharge will be maximum for constant values of C and i, when $\frac{A^3}{P}$ is maximum. $\frac{A^3}{P}$ will be maximum when $\frac{d}{d\theta}\left(\frac{A^3}{P}\right) = 0$.

Differentiating this equation with respect to θ and equation the same to zero, we get

$$\frac{P \times 3A^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta}}{P^2} = 0 \quad \text{or} \quad 3PA^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta} = 0$$

Dividing by A^2, $3P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$ ...(*i*)

But from equation (16.16), $P = 2R\theta$

$$\therefore \quad \frac{dP}{d\theta} = 2R$$

But, P = 2Rø,

So dP/dR = 2ø

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$\frac{dA}{d\theta} = R^2\,(1 - \cos 2\theta)$$

Substituting the values of P, A, $\frac{dP}{d\theta}$ and $\frac{dA}{d\theta}$ in equation (*i*)

$$3 \times 2R\theta \times R^2 (1 - \cos 2\theta) - R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times 2R = 0$$

$$6R^3\theta (1 - \cos 2\theta) - 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

Dividing by $2R^3$, we get

$$3\theta (1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0 \quad \text{or} \quad 3\theta - 3\theta \cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

or

$$2\theta - 3\theta \cos 2\theta + \frac{\sin 2\theta}{2} = 0 \quad \text{or} \quad 4\theta - 6\theta \cos 2\theta + \sin 2\theta = 0$$

The solution of this equation by hit and trial, gives

$$2\theta = 308°$$

$$\therefore \quad \theta = \frac{308°}{2} = 154°$$

Depth of Flow for maximum discharge:

$$d = OD - OC = R - R\cos\theta$$
$$= R[1 - \cos\theta] = R[1 - \cos 154°]$$
$$= R[1 - \cos(180° - 26°)] = R[1 + \cos 26°] = 1.898\ R$$
$$= 1.898 \times \frac{D}{2} = 0.948\ D \simeq 0.95\ D$$

Where D = Diameter of the circular channel

Thus for maximum discharge through a circular channel the depth of flow is equal to 0.95 times its diameter.

Note:

Students you are advised to try the numericals being told & explained in the class.

UNIT-4

Hydraulic machines are defined as those machines which convert either hydraulic energy (energy possessed by water) into mechanical energy (which is further converted into electrical energy) or mechanical energy into hydraulic energy. The hydraulic machines, which convert the hydraulic energy into mechanical energy, are called turbines while the hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps. Thus, the study of hydraulic machines consists of study of turbines and pumps. Turbines consists of mainly study of Pelton turbine, Francis Turbine and Kaplan Turbine while pumps consist of study of centrifugal pump and reciprocating pumps.

4.1 TURBINES

Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy. This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus, the mechanical energy is converted into electrical energy. The electric power which is obtained from the hydraulic energy (energy of water) is known as Hydroelectric power.

At present the generation of hydroelectric power is the cheapest as compared by the power generated by other sources such as oil, coal etc.

4.2 GENERAL LAYOUT OF A HYDROELECTRIC POWER PLANT

Fig. below shows a general layout of a hydroelectric power plant which consists of:

- A dam constructed across a river to store water.
- Pipes of large diameters called penstocks, which carry water under pressure from the storage reservoir to the turbines. These pipes are made of steel or reinforced concrete.
- Turbines having different types of vanes fitted to the wheels.
- Tail race, which is a channel which carries water away from the turbines after the water has worked on the turbines. The surface of water in the tail race channel is also known as tail race.

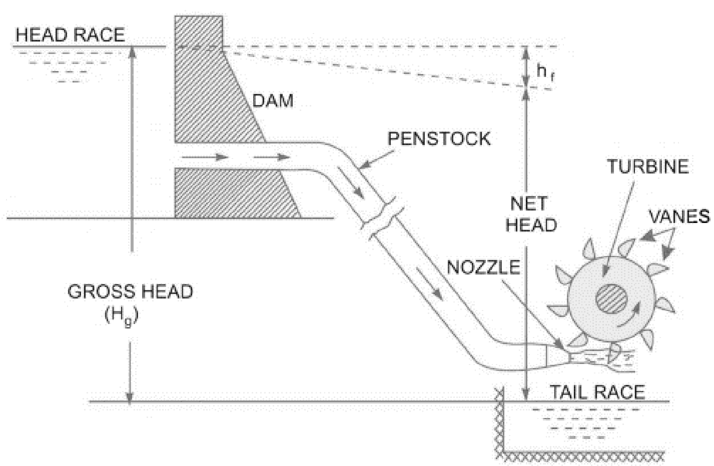


Fig. 4.1 Layout of Hydroelectric Power Plant

- Gross Head: The difference between the head race level and tail race level when no water is flowing is known as Gross Head.
- Net Head: It is also called effective head and is defined as the head available at the inlet of the turbine. When water is flowing from head race to the turbine, a loss of head due to friction between the water and penstocks occurs. Though there are other losses also such as loss due to bend, pipe fittings, loss at the entrance of penstock

etc., yet they are having small magnitude as compared to head loss due to friction. If 'h/is the head loss due to friction between penstocks and water then net heat on turbine is given by $H = H_g – h_f$.

Where H_g = Gross Head and $h_f = 4×f×l×V^2 / D × 2g$, in which V = Velocity of flow in penstock,

L = Length of penstock, D = Diameter of penstock.

4.3 Efficiency of turbines:

- **Hydraulic Efficiency:** It is defined as the ratio of power given by water to the runner of a turbine (runner is a rotating part of a turbine and on the runner vanes are fixed) to the power supplied by the water at the inlet of the turbine. The power at the inlet of the turbine is more and this power goes on decreasing as the water flows over the vanes of the turbine due to hydraulic losses as the vanes are not smooth. Hence, the power delivered to the runner of the turbine will be less than the power available at the inlet of the turbine. Thus, mathematically, the hydraulic efficiency of a turbine is written as

  "Power Delivered to runner/ Power supplied to inlet"

where R.P. = Power delivered to runner *i.e.*, runner power

$$= \frac{W}{g}\frac{\left[V_{w_1} \pm V_{w_2}\right] \times u}{1000} \text{ kW} \qquad \text{...for Pelton Turbine}$$

$$= \frac{W}{g}\frac{\left[V_{w_1}u_1 \pm V_{w_2}u_2\right]}{1000} \text{ kW} \qquad \text{...for a radial flow turbine}$$

W.P. = Power supplied at inlet of turbine and also called water power

$$= \frac{W \times H}{1000} \text{ kW} \qquad \text{...(18.3)}$$

where W = Weight of water striking the vanes of the turbine per second

$= \rho g \times Q$ in which Q = Volume of water/s,

V_{w_1} = Velocity of whirl at inlet,

V_{w_2} = Velocity of whirl at outlet,

u = Tangential velocity of vane,

u_1 = Tangential velocity of vane at inlet for radial vane,

u_2 = Tangential velocity of vane at outlet for radial vane,

H = Net head on the turbine.

Power supplied at the inlet of turbine in S.I.units is known as water power. It is given by

$$\text{W.P.} = \frac{\rho \times g \times Q \times H}{1000} \text{ kW}$$

For water $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad \text{W.P.} = \frac{1000 \times g \times Q \times H}{1000} = g \times Q \times H \text{ kW}$$

- **Mechanical Efficiency:** The power delivered by water to the runner of a turbine is transmitted to the shaft of the turbine. Due to mechanical losses, the power available at the shaft of the turbine is less than the power delivered to the runner of a turbine. The ratio of the power available at the shaft of the turbine (known as S.P. or B.P.) to the power delivered to the runner is defined as mechanical efficiency. Hence, mathematically, it is written as

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner}} = \frac{\text{S.P.}}{\text{R.P.}}$$

- **Volumetric Efficiency:** The volume of the water striking the runner of a turbine is slightly less than the volume of the water supplied to the turbine. Some of the volume of the water is discharged to the tail race without striking the runner of the turbine. Thus the ratio of the volume of the water actually striking the runner to the volume of water supplied to the turbine is defined as volumetric efficiency. It is written as

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}}$$

- **Overall Efficiency:** It is defined as the ratio of power available at the shaft of the turbine to the power supplied by the water at the inlet of the turbine. It is written as:

$$\eta_o = \frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}} \times \frac{\text{R.P.}}{\text{R.P.}} \qquad \text{(where R.P. = Power delivered to runner)}$$

$$= \frac{\text{S.P.}}{\text{R.P.}} \times \frac{\text{R.P.}}{\text{W.P.}}$$

If the shaft power is taken in kW then water power should also be taken in kW. Shaft power is commonly represented by P.

$$\text{Water power in kW} = \frac{\rho \times g \times Q \times H}{1000}, \text{ where } \rho = 1000 \text{ kg/m}^3$$

$$\therefore \qquad \eta_o = \frac{\text{Shaft power in kW}}{\text{Water power in kW}} = \frac{P}{\left(\frac{\rho \times g \times Q \times H}{1000}\right)}$$

where P = Shaft power.

4.4 CLASSIFICATION OF TURBINES:

The hydraulic turbines are classified according to the type of energy available at the inlet of the turbine, direction of flow through the vanes, head at the inlet of the turbine and specific speed of the turbines. Thus the following are the important classifications of the turbines:

1. According to the type of energy at inlet:
   a. Impulse turbine, and
   b. Reaction turbine.
2. According to the direction of flow through runner:
   a. Tangential flow turbine,
   b. Radial flow turbine,
   c. Axial flow turbine, and
   d. Mixed flow turbine.
3. According to the head at the inlet of turbine:
   a. High head turbine,
   b. Medium head turbine, and
   c. Low head turbine.
4. According to the specific speed of the turbine:
   a. Low specific speed turbine,
   b. Medium specific speed turbine, and
   c. High specific speed turbine.

If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as impulse turbine. As the water flows over the vanes, the pressure is atmospheric from inlet to outlet of the turbine. If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as reaction turbine. As the waters flows through the runner, the water is under pressure and the pressure energy goes on changing into kinetic energy. The runner is completely enclosed in an air-tight casing and the runner and casing is completely full of water.

If the water flows along the tangent of the runner, the turbine is known as tangential flow turbine. If the water flows in the radial direction through the runner, the turbine is called radial flow turbine. If the water flows from outwards to inwards, radially, the turbine is known as inward radial flow turbine, on the other hand, if water flows radially from inwards to outwards, the turbine is known as outward radial flow turbine. If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called axial flow turbine. If the water flows through the runner in the radial direction but leaves in the direction parallel to axis of rotation of the runner, the turbine is called mixed flow turbine.

4.5 PELTON WHEEL TURBINE

The Pelton wheel or Pelton turbine is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmospheric. This turbine is used for high heads and is named after L.A. Pelton, an American Engineer.

The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted. The nozzle increases the kinetic energy of the water flowing through the penstock. At the outlet of the nozzle, the water comes out in the form of a jet and strikes the buckets (vanes) of the runner. The main parts of the Pelton turbine are:

1. Nozzle and flow regulating arrangement (spear),
2. Runner and buckets,
3. Casing, and
4. Breaking jet.

Nozzle and Flow Regulating Arrangement: The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle as shown in Fig. below. The spear is a conical needle which is operated either by a hand wheel or automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.

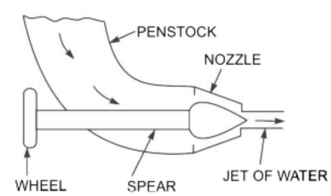


Fig. 4.2 Nozzle Arrangement

Runner with Buckets: Fig. below shows the runner of a Pelton wheel. It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts by a dividing wall which is known as splitter. The jet of water strikes on the splitter. The splitter divides the jet into two equal parts and the jet comes out at the outer edge of the bucket. The buckets are shaped in such a way that the jet gets deflected through 160° or 170°. The buckets are made of cast iron, cast steel bronze or stainless steel depending upon the head at the inlet of the turbine.

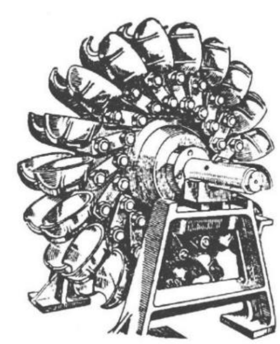


Fig. 4.3 Runner of Pelton

Casing: Figure below shows a Pelton turbine with a casing. The function of the casing is to prevent the splashing of the water and to discharge water to tail race. It also acts as safeguard against accidents. It is made of cast iron or fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

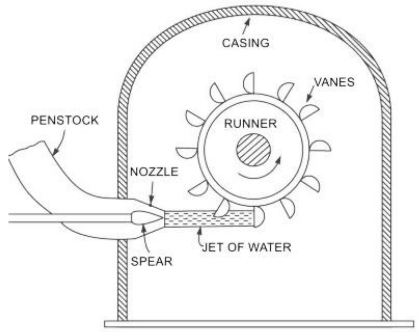


Fig. 4.4 Casing of Pelton Wheel

Breaking Jet: When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of the vanes. This jet of water is called breaking jet.

4.6 FRANCIS TURBINE:

The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine, after the name of J.B. Francis, an American engineer who in the beginning designed inward radial flow reaction type of turbine. In the modern Francis turbine, the water enters the runner of the turbine in the radial direction at outlet and leaves in the axial direction at the inlet of the runner. Thus the modern Francis Turbine is a mixed flow type turbine.

The velocity triangle at inlet and outlet of the Francis turbine are drawn in the same way as in case of inward flow reaction turbine. As in case of Francis turbine, the discharge is radial at outlet, the velocity of whirl at outlet (i.e., V w) will be zero. Hence the work done by water on the runner per second will be

$$= pQ[V_{w1}u_1]$$

And work done per second per unit weight of water striking/s = $1/g\ [V_{w1}u_1]$

Hydraulic efficiency will be given by, $\eta_h = \frac{V_{w_1} u_1}{gH}$.

The following are the important relations for Francis Turbines :

1. The ratio of width of the wheel to its diameter is given as $n = B_1/D_1$. The value of n varies from 0.10 to .40.
2. The flow ratio is given as, Flow ratio= $\frac{V_{f_1}}{\sqrt{2gH}}$ and varies from 0.15 to 0.30.

The speed ratio= $\frac{u_1}{\sqrt{2gH}}$ varies from 0.6 to 0.9.

4.7 CENTRIFUGAL PUMPS

The hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps. The hydraulic energy is in the form of pressure energy. If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

The centrifugal pump acts as a reverse of an inward radial flow reaction turbine. This means that the flow in centrifugal pumps is in the radial outward directions. The centrifugal pump works on the principle of forced vortex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place. The rise in pressure head at any point of the rotating liquid is proportional to the square of tangential velocity of the liquid at that point. Thus, at the outlet of the impeller, where radius is more, the rise in pressure head will be more and the liquid will be discharged at the outlet with a high-pressure head. Due to this high-pressure head, the liquid can be lifted to a high level.

4.7.1 MAIN PARTS OF A CENTRIFUGAL PUMP

The following are the main parts of a centrifugal pump:

- Impeller.
- Casing.
- Suction pipe with a foot valve and a strainer.
- Delivery pipe.

All the main parts of the centrifugal pump are shown in Fig. below:

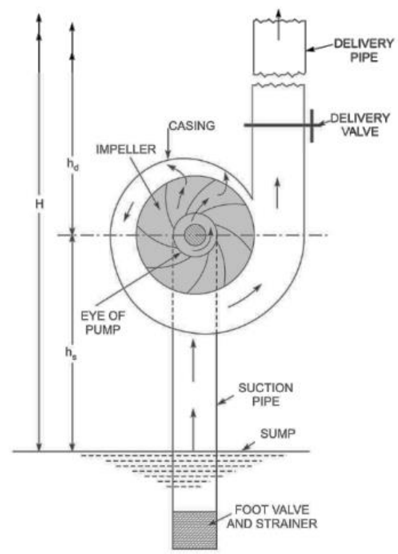


Fig. 4.5 Centrifugal Pump

1. Impeller. The rotating part of a centrifugal pump is called 'impeller'. It consists of a series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.
2. Casing. The casing of a centrifugal pump is similar to the casing of a reaction turbine. It is an air-tight passage surrounding the impeller and is designed in such a way that the kinetic energy of the water discharged at the outlet of the impeller is converted into pressure energy before the water leaves the casing and enters the delivery pipe. The following three types of the casings are commonly adopted:
    a. Volute Casing
    b. Vortex Casing
    c. Casing with guide blades
3. Volute Casing: Fig below shows the volute casing, which surrounds the impeller. It is of spiral type in which area of flow increases gradually. The increase in area of flow decreases the velocity of flow. The decrease in velocity increases the pressure of the water flowing through the casing. It has been observed that in case of volute casing, the efficiency of the pump increases slightly as a large amount of energy is lost due to the formation of eddies in this type of casing.
4. Vortex Casing: If a circular chamber is introduced between the casing and the impeller as shown in Fig, the casing is known as Vortex Casing. By introducing the circular chamber, the loss of energy due to the formation of eddies is reduced to a considerable extent. Thus the efficiency of the pump is more than the efficiency when only volute casing is provided.
5. Casing with Guide Blades: This casing is shown in Fig, in which the impeller is surrounded by a series of guide blades mounted on a ring which is known as diffuser. The guide vanes are designed in such a way that the water from the impeller enters the guide vanes without stock. Also the area of the guide vanes increases, thus reducing the velocity of flow through guide vanes and consequently increasing the pressure of water. The water from the guide vanes then passes through the surrounding casing which is in most of the cases concentric with the impeller.

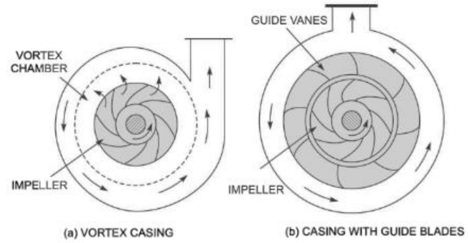


Fig. 4.6 Different type of Casing

6. Suction Pipe with a Foot valve and a Strainer: A pipe whose one end is connected to the inlet of the pump and other end dips into water in a sump is known as suction pipe. A foot valve which is a non-return valve or one-way type of valve is fitted at the lower end of the suction pipe. The foot valve opens only in the upward direction. A strainer is also fitted at the lower end of the suction pipe.
7. Delivery Pipe. A pipe whose one end is connected to the outlet of the pump and other end delivers the water at a required height is known as delivery pipe.

4.7.2 ANALYSIS OF FLOW THROUGH THE IMPELLAR OR WORKING OF C.P

In runners of turbines and in impellers of pumps, the flow of fluid is usually a combination of: (a) circulatory flow (i.e., flow in concentric circles), and (b) radial flow (i.e., flow involving a change of distance from the axis of rotation). The path resulting from the superimposition of these two motions is in the form of a spiral. In a centrifugal pump, water enters through an opening provided at the centre and leaves at the periphery. Guide vanes are made of spiral shape to enable water to have both circulatory and radial flows inside the impeller.

As the radius increases from inlet to outlet, the area across the flow must also increase, and consequently the relative velocity decreases so that $v_1 = \frac{q}{a_1}$ and $v_2 = \frac{q}{a_2}$ where q = quantity of water flowing per second, and a_1, a_2 are areas across flow at the inlet and outlet, respectively between two consecutive blades.

Therefore, pressure difference due to the change of kinetic energy is given as $\frac{v_1^2 - v_2^2}{2g}$ per unit weight of water. Also, pressure difference (ΔP) due to the centrifugal head can be expressed as: $\frac{\omega^2}{2g}(R_2^2 - R_1^2)$, where, R_1 and R_2 are inner radius and outer radius of the impeller, respectively. This pressure difference is due to cylindrical vortex only.

Therefore, the total pressure difference due to two flows (i.e., circulatory and radial) or due to spiral vortex will be:

$$\frac{p_2 - p_1}{\gamma} = \frac{\omega^2}{2g}\left(R_2^2 - R_1^2\right) + \frac{v_1^2 - v_2^2}{2g} \quad \Rightarrow \quad H = \frac{u_2^2 - u_1^2}{2g} + \frac{v_1^2 - v_2^2}{2g}$$

Where, H = head developed by the impeller, u_1 = peripheral velocity of water at the inlet of the impeller, and u_2 = peripheral velocity of water at the outlet of the impeller.

4.7.3 HEADS & EFFICIENCY OF CENTRIFUGAL PUMP

4.7.3.1 Heads

1. Suction Head: It is the vertical height of the centre line of the C.P above the water surface in the tank or pump from which water is to be lifted. This height is also called suction lift & denoted by 'h_s'.
2. Delivery Head (h_d). The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head. This is denoted by 'h_d'.

3. Static Head (H_s). The sum of suction head and delivery head is known as static head. This is represented by 'H_s' and is written as $H_s = h_s + h_d$
4. Manometric Head (H_m): The manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by 'H_m'. It is given by the following expressions:

(*a*) H_m = Head imparted by the impeller to the water – Loss of head in the pump

$$= \frac{V_{w_2} u_2}{g} - \text{Loss of head in impeller and casing} \quad ...(19.4)$$

$$= \frac{V_{w_2} u_2}{g} \text{ ...if loss of pump is zero} \quad ...(19.5)$$

(*b*) H_m = Total head at outlet of the pump – Total head at the inlet of the pump

$$= \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right) \quad ...(19.6)$$

where $\frac{p_o}{\rho g}$ = Pressure head at outlet of the pump = h_d

$\frac{V_o^2}{2g}$ = Velocity head at outlet of the pump

= Velocity head in delivery pipe = $\frac{V_d^2}{2g}$

Z_o = Vertical height of the outlet of the pump from datum line, and

$\frac{p_i}{\rho g}, \frac{V_i^2}{2g}$, Z_i = Corresponding values of pressure head, velocity head and datum head at the inlet of the pump,

i.e., h_s, $\frac{V_s^2}{2g}$ and Z_s respectively.

(*c*)

$$H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g} \quad ...(19.7)$$

where h_s = Suction head, h_d = Delivery head,
h_{f_s} = Frictional head loss in suction pipe, h_{f_d} = Frictional head loss in delivery pipe, and
V_d = Velocity of water in delivery pipe.

4.7.3.2 EFFICIENCIES:

Efficiencies of a Centrifugal Pump. In case of a centrifugal pump, the power is transmitted from the shaft of the electric motor to the shaft of the pump and then to the impeller. From the impeller, the power is given to the water. Thus power is decreasing from the shaft of the pump to the impeller and then to the water. The following are the important efficiencies of a centrifugal pump:

- **Manometric efficiency:** The ratio of the manometric head to the head imparted by the impeller to the water is known as manometric efficiency. Mathematically, it is written as

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$= \frac{H_m}{\left(\frac{V_{w_2} u_2}{g}\right)} = \frac{g H_m}{V_{w_2} u_2}$$

The power at the impeller of the pump is more than the power given to the water at outlet of the pump. The ratio of the power given to water at outlet of the pump to the power available at the impeller, is known as manometric efficiency.

The power given to water at outlet of the pump = $\frac{WH_m}{1000}$ kW

$$\text{The power at the impeller} = \frac{\text{Work done by impeller per second}}{1000} \text{kW}$$

$$= \frac{W}{g} \times \frac{V_{w_2} \times u_2}{1000} \text{kW}$$

$$\eta_{man} = \frac{\frac{W \times H_m}{1000}}{\frac{W}{g} \times \frac{V_{w_2} \times u_2}{1000}} = \frac{g \times H_m}{V_{w_2} \times u_2}.$$

- **Mechanical efficiency:** The power at the shaft of the centrifugal pump is more than the power available at the impeller of the pump. The ratio of the power available at the impeller to the power at the shaft of the centrifugal pump is known as mechanical efficiency. It is written as

$$\eta_m = \frac{\text{Power at the impeller}}{\text{Power at the shaft}}$$

$$\text{The power at the impeller in kW} = \frac{\text{Work done by impeller per second}}{1000}$$

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000} \quad \text{[Using equation (19.2)]}$$

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} u_2}{1000}\right)}{\text{S.P.}} \quad \text{...(19.9)}$$

where S.P. = Shaft power.

- **Overall efficiency:** It is defined as ratio of power output of the pump to the power input to the pump. The power output of the pump in Kw.

$$= \frac{\text{Weight of water lifted} \times H_m}{1000} = \frac{WH_m}{1000}$$

Power input to the pump = Power supplied by the electric motor

= S.P. of the pump.

$$\therefore \quad \eta_o = \frac{\left(\frac{WH_m}{1000}\right)}{\text{S.P.}}$$

$$\text{Also} \quad \eta_o = \eta_{man} \times \eta_m.$$

4.8 PRIMING OF A CENTRIFUGAL PUMP

Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe upto the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump. Thus the air from these parts of the pump is removed and these parts are filled with the liquid to be pumped.

The work done by the impeller per unit weight of liquid per sec is known as the head generated by the pump.

Equation gives the head generated by the pump as= $1/g[V_{w2}u_2]$ metre. This equation is independent of the density of the liquid. This means that when pump is running in air, the head generated is in terms of metre of air. If the pump is primed with water, the head generated is same metre of water. But as the density of air is very low, the generated head of air in terms of equivalent metre of water head is negligible and hence the water may not be sucked from the pump. To avoid this difficulty, priming is necessary.

4.9 CAVITATION:

Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which these vapour bubbles collapse, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the metallic surface and also considerable noise and vibrations are produced.

Cavitation includes formation of vapour bubbles of the flowing liquid and collapsing of the vapour bubbles. Formation of vapour bubbles of the flowing liquid take place only whenever the pressure in any region falls below vapour pressure. When the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling and vapour bubbles are formed. These vapour bubbles are carried along with the flowing liquid to higher pressure zones where these vapours condense and bubbles collapse. Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced and metallic surfaces are subjected to high local stresses. Thus the surfaces are damaged.

4.9.1 Precaution Against Cavitation:

The following precautions should be taken against cavitation :

- The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water, then the absolute pressure head should not be below 2.5 m of water.
- The special materials or coatings such as aluminium-bronze and stainless steel, which are cavitation resistant materials, should be used.

4.9.2 Effects of Cavitation:

The following are the effects of cavitation :

- The metallic surfaces are damaged and cavities are formed on the surfaces.
- Due to sudden collapse of vapour bubble, considerable noise and vibrations are produced.
- The efficiency of a turbine decreases due to cavitation. Due to pitting action, the surface of the turbine blades becomes rough and the force exerted by water on the turbine blades decreases. Hence, the work done by water or output horse power becomes less and thus efficiency decreases.

4.9.3 Hydraulic Machines Subjected to Cavitation:

The hydraulic machines subjected to cavitation are reaction turbines and centrifugal pumps.

4.9.3.1 Cavitation in Turbines:

In turbines, only reaction turbines are subjected to cavitation. In reaction turbines the cavitation may occur at the outlet of the runner or at the inlet of the draft tube where the pressure is considerably reduced (i.e., which may be below the vapour pressure of the liquid flowing through the turbine). Due to cavitation, the metal of the runner vanes and draft-tube is gradually eaten away, which results in lowering the efficiency of the turbine. Hence, the cavitation in a reaction turbine can be noted by a sudden drop in efficiency. In order to determine whether cavitation will occur in any portion of a reaction turbine, the critical value of Thoma's cavitation factor (sigma) is calculated.

4.9.3.2 Cavitation in Centrifugal Pumps:

In centrifugal pumps the cavitation may occur at the inlet of the impeller of the pump, or at the suction side of the pumps, where the pressure is considerably reduced. Hence if the pressure at the suction side of the pump drops below the vapour pressure of the liquid then the cavitation may occur. The cavitation in a pump can be noted by a sudden drop in efficiency and head. In order to determine whether cavitation will occur in any portion of the suction side of the pump, the critical value of Thoma's cavitation factor is calculated.

NUMERICALS

Problem 16.1 *Find the velocity of flow and rate of flow of water through a rectangular channel of 6 m wide and 3 m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take Chezy's constant C = 55.*

Solution. Given :

Width of rectangular channel, $b = 6$ m

Depth of channel, $d = 3$ m

∴ Area, $A = 6 \times 3 = 18 \text{ m}^2$

Bed slope, $i = 1 \text{ in } 2000 = \frac{1}{2000}$

Chezy's constant, $C = 55$

Perimeter, $P = b + 2d = 6 + 2 \times 3 = 12$ m

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{18}{12} = 1.5$ m

Velocity of flow is given by equation (16.4) as,

$$V = C\sqrt{mi} = 55\sqrt{1.5 \times \frac{1}{2000}} = \mathbf{1.506 \text{ m/s. Ans.}}$$

Rate of flow, $Q = V \times \text{Area} = V \times A = 1.506 \times 18 = \mathbf{27.108 \text{ m}^3\text{/s. Ans.}}$

Problem 16.2 *Find the slope of the bed of a rectangular channel of width 5 m when depth of water is 2 m and rate of flow is given as 20 m³/s. Take Chezy's constant, C = 50.*

Solution. Given :

Width of channel, $b = 5$ m

Depth of water, $d = 2$ m

Rate of flow, $Q = 20 \text{ m}^3\text{/s}$

Chezy's constant $C = 50$

Let the bed slope $= i$

Using equation (16.5), we have $Q = AC\sqrt{mi}$

where $A = \text{Area} = b \times d = 5 \times 2 = 10 \text{ m}^2$

$$m = \frac{A}{P} = \frac{10}{b + 2d} = \frac{10}{5 + 2 \times 2} = \frac{10}{5 + 4} = \frac{10}{9} \text{ m}$$

$$\therefore \quad 20.0 = 10 \times 50 \times \sqrt{\frac{10}{9} \times i} \text{ or } \sqrt{\frac{10}{9} i} = \frac{20.0}{500} = \frac{2}{50}$$

Squaring both sides, we have $\frac{10}{9} i = \frac{4}{2500}$

$$\therefore \quad i = \frac{4}{2500} \times \frac{9}{10} = \frac{36}{25000} = \frac{1}{\frac{25000}{36}} = \frac{\mathbf{1}}{\mathbf{694.44}}\text{. }\mathbf{Ans.}$$

∴ Bed slope is 1 in 694.44.

Problem 16.3 *A flow of water of 100 litres per second flows down in a rectangular flume of width 600 mm and having adjustable bottom slope. If Chezy's constant C is 56, find the bottom slope necessary for uniform flow with a depth of flow of 300 mm. Also find the conveyance K of the flume.*

Solution. Given :

Discharge, $Q = 100 \text{ litres/s} = \frac{100}{1000} = 0.10 \text{ m}^3/\text{s}$

Width of channel, $b = 600 \text{ mm} = 0.60 \text{ m}$

Depth of flow, $d = 300 \text{ mm} = 0.30 \text{ m}$

∴ Area of flow, $A = b \times d = 0.6 \times 0.3 = 0.18 \text{ m}^2$

Chezy's constant, $C = 56$

Let the slope of bed $= i$

Hydraulic mean depth, $m = \frac{A}{P} = \frac{0.18}{b + 2d} = \frac{0.18}{0.6 + 2 \times 0.30} = \frac{0.18}{1.2} = 0.15 \text{ m}$

Using equation (16.5), we have $Q = AC\sqrt{mi}$

or $$0.10 = 0.18 \times 56 \times \sqrt{0.15 \times i} \quad \text{or} \quad \sqrt{0.15i} = \frac{0.10}{0.18 \times 56}$$

Squaring both sides, we have $0.15\, i = \left(\frac{0.10}{0.18 \times 56}\right)^2 = .000098418$

∴ $$i = \frac{.000098418}{0.15} = .0006512 = \frac{1}{\frac{1}{.0006512}} = \frac{1}{1524} \text{. Ans.}$$

∴ Slope of the bed is 1 in 1524.

Conveyance K of the channel

Equation (16.5) is given as $Q = AC\sqrt{mi}$

which can be written as $Q = K\sqrt{i}$

where $K = AC\sqrt{m}$ and K is called conveyance of the channel section.

∴ $$K = AC\sqrt{m} = 0.18 \times 56 \times \sqrt{0.15} = \mathbf{3.9039 \text{ m}^3/\text{s}}\text{. Ans.}$$

Problem 16.4 *Find the discharge through a trapezoidal channel of width 8 m and side slope of 1 horizontal to 3 vertical. The depth of flow of water is 2.4 m and value of Chezy's constant, C = 50. The slope of the bed of the channel is given 1 in 4000.*

Solution. Given :

Width, $b = 8 \text{ m}$

Side slope $= 1$ hor. to 3 vertical

Depth, $d = 2.4 \text{ m}$

Chezy's constant, $C = 50$

Bed slope, $i = \frac{1}{4000}$

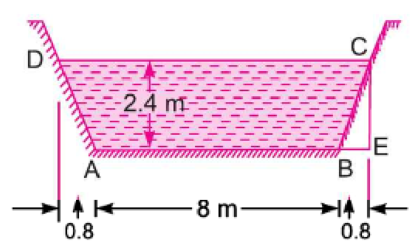


Fig. 16.3

From Fig. 16.3 when depth , $CE = 2.4$,

the horizontal distance $BE = 2.4 \times \frac{1}{3} = 0.8$ m

∴ Top width of the channel,

$$CD = AB + 2 \times BE = 8.0 + 2 \times 0.8 = 9.6 \text{ m}$$

∴ Area of trapezoidal channel, $ABCD$ is given as,

$$A = (AB + CD) \times \frac{CE}{2} = (8 + 9.6) \times \frac{2.4}{2} = 17.6 \times 1.2 = 21.12 \text{ m}^2$$

Wetted perimeter, $P = AB + BC + AD = AB + 2BC$ $(\because \ BC = AD)$

But $BC = \sqrt{BE^2 + CE^2} = \sqrt{(0.8)^2 + (2.4)^2} = 2.529$ m

∴ $P = 8.0 + 2 \times 2.529 = 13.058$ m

Hydraulic mean depth, $m = \frac{A}{P} = \frac{12.12}{13.058} = 1.617$ m

The discharge, Q is given by equation (16.5) as

$$Q = AC\sqrt{mi} = 21.12 \times 50\sqrt{1.617 \times \frac{1}{4000}} = \mathbf{21.23 \ m^3/s.\ Ans.}$$

Problem 16.5 *Find the bed slope of trapezoidal channel of bed width 6 m, depth of water 3 m and side slope of 3 horizontal to 4 vertical, when the discharge through the channel is 30 m³/s. Take Chezy's constant, C = 70.*

Solution. Given :

Bed width, $b = 6.0$ m

Depth of flow, $d = 3.0$ m

Side slope = 3 horizontal to 4 vertical

Discharge, $Q = 30 \text{ m}^3/\text{s}$

Chezy's constant, $C = 70$

From Fig. 16.4, for depth of flow = 3 m = CE

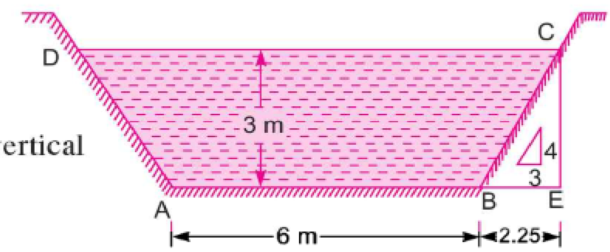


Fig. 16.4

Distance, $BE = 3 \times \frac{3}{4} = \frac{9}{4} = 2.25$ m

∴ Top width, $CD = AB + 2 \times BE = 6.0 + 2 \times 2.25 = 10.50$ m

Wetted perimeter, $P = AD + AB + BC = AP + 2BC$ $(\because BC = AD)$

$$= AB + 2\sqrt{BE^2 + CE^2} = 6.0 + 2\sqrt{(2.25)^2 + (3)^2} = 13.5 \text{ m}$$

Area of flow, A = Area of trapezoidal $ABCD$

$$= \frac{(AB + CD) \times CE}{2} = \frac{(6 + 10.50)}{2} \times 3.0 = 24.75 \text{ m}^2$$

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{24.75}{13.50} = 1.833$

Using equation (16.5), $Q = AC\sqrt{mi}$

or $30.0 = 24.75 \times 70 \times \sqrt{1.833 \times i} = 2345.6\sqrt{i}$

$$i = \left(\frac{30}{2345.6}\right)^2 = \frac{1}{\left(\frac{2345.6}{30}\right)^2} = \mathbf{\frac{1}{6133}}\textbf{. Ans.}$$

Problem 16.6 *Find the discharge of water through the channel shown in Fig. 16.5. Take the value of Chezy's constant = 60 and slope of the bed as 1 in 2000.*

Solution. Given :

Chezy's constant, $C = 60$

Bed slope, $i = \frac{1}{2000}$

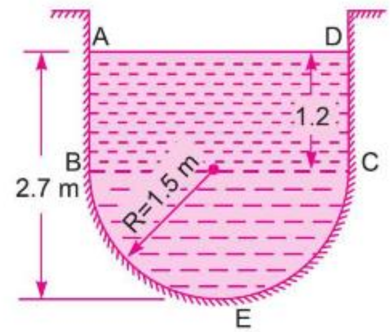


Fig. 16.5

From Fig.16.5, Area, $A = \text{Area } ABCD + \text{Area } BEC$

$$= (1.2 \times 3.0) + \frac{\pi R^2}{2}$$

$$= 3.6 + \frac{(1.5)^2}{2} = 7.134 \text{ m}^2$$

Wetted perimeter, $P = AB + BEC + CD$

$$= 1.2 + \pi R + 1.2 = 1.2 + \pi \times 1.5 + 1.2 = 7.1124 \text{ m}$$

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{7.134}{7.1124} = 1.003$

The discharge, Q is given by equation (16.5) as

$$Q = AC\sqrt{mi}$$

$$= 7.134 \times 60 \times \sqrt{1.003 \times \frac{1}{2000}} = \mathbf{9.585\ m^3/s.\ Ans.}$$

Problem 16.7 *Find the rate of flow of water through a V-shaped channel as shown in Fig. 16.6. Take the value of C = 55 and slope of the bed 1 in 2000.*

Solution. Given :

Chezy's constant, $C = 55$

Bed slope, $i = \frac{1}{1000}$

Depth of flow, $d = 4.0$ m

Angle made by each side with vertical,

i.e., $\angle ABD = \angle CBD = 30°$

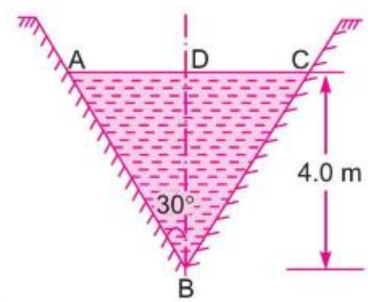


Fig. 16.6

From Fig. 16.6, we have

Area, $A = \text{Area of } ABC$

$$= 2 \times \text{Area } ABD = \frac{2 \times AD \times BD}{2} = AD \times BD$$

$$= BD \tan 30° \times BD \qquad \left(\because \tan 30° = \frac{AD}{BD}, AD = BD \tan 30°\right)$$

$$= 4 \tan 30° \times 4 = 9.2376 \text{ m}^2$$

Wetted perimeter, $P = AB + BC = 2AB$ $(\because \; AB = BC)$

$$= 2\sqrt{BD^2 + AD^2} = 2\sqrt{4.0^2 + (4 \tan 30°)^2}$$

$$= 2\sqrt{16.0 + 5.333} = 9.2375 \text{ m.}$$

$\therefore$ Hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{9.2376}{9.2375} = 1.0$ m

Using equation (16.5) for discharge,

$$Q = AC\sqrt{mi} = 9.2376 \times 55 \times \sqrt{1 \times \frac{1}{1000}} = \mathbf{16.066\ m^3/s.\ Ans.}$$

1. **Bazin formula** (In MKS units) : $C = \dfrac{157.6}{1.81 + \dfrac{K}{\sqrt{m}}}$ ...(16.6)

where K = Bazin's constant and depends upon the roughness of the surface of channel, whose values are given in Table 16.1.
m = Hydraulic mean depth or hydraulic radius.

2. **Ganguillet-Kutter Formula.** The value of C is given in MKS unit as

$$C = \frac{23 + \dfrac{0.00155}{i} + \dfrac{1}{N}}{1 + \left(23 + \dfrac{0.00155}{i}\right)\dfrac{N}{\sqrt{m}}} \quad ...(16.7)$$

where N = Roughness co-efficient which is known as Kutter's constant, whose value for different surfaces are given in Table 16.2
i = Slope of the bed
m = Hydraulic mean depth.

3. **Manning's Formula.** The value of C according to this formula is given as

$$C = \frac{1}{N} m^{1/6} \quad ...(16.8)$$

where m = Hydraulic mean depth
N = Manning's constant which is having same value as Kutter's constant for the normal range of slope and hydraulic mean depth. The values of N are given in Table 16.2.

S. No.	Nature of Channel inside surface	Value of N
1.	Very smooth surface of glass, plastic or brass	0.010
2.	Smooth surface of concrete	0.012
3.	Rubble masonary or poor brick work	0.017
4.	Earthen channels neatly excavated	0.018
5.	Earthen channels of ordinary surface	0.027
6.	Earthen channels of rough surface	0.030
7.	Natural streams, clean and straight	0.030
8.	Natural streams with weeds, duppools etc.	0.075 to .15

Problem 16.8 *Find the discharge through a rectangular channel 2.5 m wide, having depth of water 1.5 m and bed slope as 1 in 2000. Take the value of k = 2.36 in Bazin's formula.*

Solution. Given :

Width of channel, $b = 2.5$ m

Depth of flow, $d = 1.5$ m

∴ Area, $A = b \times d = 2.5 \times 1.5 = 3.75 \text{ m}^2$

Wetted Perimeter, $P = d + b + d = 1.5 + 2.5 + 1.5 = 5.5$ m

∴ Hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{3.75}{5.50} = 0.682$

Bed slope, $i = \dfrac{1}{2000}$

Bazin's constant, $K = 2.36$

Using Bazin's formula given by equation (16.6), as

$$C = \frac{157.6}{1.81 + \dfrac{K}{\sqrt{m}}} = \frac{157.6}{1.81 + \dfrac{2.36}{\sqrt{0.682}}} = 33.76$$

Discharge, Q is given by equation (16.5), as

$$Q = AC\sqrt{mi}$$

$$= 3.75 \times 33.76 \times \sqrt{0.682 \times \frac{1}{2000}} = \mathbf{2.337 \text{ m}^3/\text{s}. \text{ Ans.}}$$

Problem 16.9 *Find the discharge through a rectangular channel 14 m wide, having depth of water 3 m and bed slope 1 in 1500. Take the value of N = 0.03 in the Kutter's formula.*

Solution. Given :

Width of channel, $b = 4$ m

Depth of water, $d = 3$ m

Bed slope, $i = \dfrac{1}{1500} = 0.000667$

Kutter's constant, $N = 0.03$

Area of flow, $A = b \times d = 4 \times 3 = 12 \text{ m}^2$

Wetted perimeter, $P = d + b + d = 3 + 4 + 3 = 10$ m

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{.12}{10} = 1.2$ m

Using Kutter's formula given by equation (16.7), as

$$C = \frac{23 + \frac{.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{.00155}{i}\right) \times \frac{N}{\sqrt{m}}} = \frac{23 + \frac{.00155}{.000667} + \frac{1}{.03}}{1 + \left(23 + \frac{.00155}{.000667}\right) \times \frac{.03}{\sqrt{1.20}}}$$

$$= \frac{23 + 2.3238 + 33.33}{1 + (23 + 2.3238) \times .03286} = \frac{58.633}{1.832} = 32.01$$

Discharge, Q is given by equation (16.5), as

$$Q = AC\sqrt{mi} = 12 \times 32.01 \times \sqrt{12 \times .000667} = \mathbf{10.867\ m^3/s.\ Ans.}$$

Problem 16.10 *Find the discharge through a rectangular channel of width 2 m, having a bed slope of 4 in 8000. The depth of flow is 1.5 m and take the value of N in Manning's formula as 0.012.*

Solution. Given :

Width of the channel, $b = 2$ m

Depth of the flow, $d = 1.5$ m

$\therefore$ Area of flow, $A = b \times d = 2 \times 1.5 = 3.0$ m^2

Wetted perimeter, $P = b + d + d = 2 + 1.5 + 1.5 = 5.0$ m

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{3.0}{5.0} = 0.6$

Bed slope , $i = 4 \text{ in } 8000 = \frac{4}{8000} = \frac{1}{2000}$

Value of N = 0.012

Using Manning's formula, given by equation (16.8), as

$$C = \frac{1}{N} m^{1/6} = \frac{1}{0.012} \times 0.6^{1/6} = 76.54$$

Discharge, Q is given by equation (16.5), as

$$Q = AC\sqrt{mi}$$

$$= 3.0 \times 76.54 \sqrt{0.6 \times \frac{1}{2000}} \text{ m}^2/\text{s} = \mathbf{3.977\ m^3/s.\ Ans.}$$

Problem 16.11 *Find the bed slope of trapezoidal channel of bed width 4 m, depth of water 3 m and side slope of 2 horizontal to 3 vertical, when the discharge through the channel is 20 m^3/s.*

Take Manning's N = 0.03 in Manning's formula $C = \frac{1}{N} m^{1/6}$.

Solution. Given :

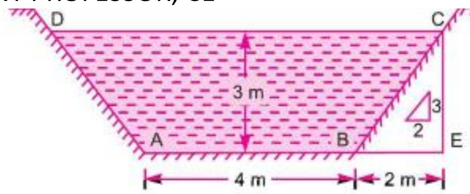


Fig. 16.7

Bed width, $b = 4$ cm

Depth of flow, $d = 3$ m

Side slope $= 2$ hor. to 3 vert.

Discharge, $Q = 20.0\ m^3/s$

Manning's, $N = 0.03$

From Fig. 16.7, we have

Distance, $BE = d \times \frac{2}{3} = 3 \times \frac{2}{3} = 2$ m

$\therefore$ Top width, $CD = AB + 2BE$

$= 4 + 2 \times 2 = 8.0$ m

$\therefore$ Area of flow, A = Area of trapezoidal section $ABCD$

$$= \frac{(AB + CD)}{2} \times d = \frac{(4+8)}{2} \times 3 = 18\ m^2$$

Wetted perimeter, $P = AD + AB + BC = AB + 2BC$ $(\because \quad AD = BC)$

$$= 4.0 + 2\sqrt{BE^2 + EC^2} = 4.0 + 2\sqrt{2^2 + 3^2} = 4.0 + 2 \times \sqrt{13} = 11.21 \text{ m}$$

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{18}{11.21} = 1.6057$

Using Manning's formula, $C = \frac{1}{N} m^{1/6} = \frac{1}{0.03} \times (1.6057)^{1/6} = 36.07$

Using equation (16.5) for discharge,

$$Q = AC\sqrt{mi} = 18 \times 36.07 \times \sqrt{1.6057 \times i} \quad \text{or } 20.0 = 822.71\sqrt{i}$$

$$\therefore \quad i = \left(\frac{20.0}{822.71}\right)^2 = 0.0005909 = \frac{1}{1692}. \textbf{ Ans.}$$

Problem 16.12 *Find the diameter of a circular sewer pipe which is laid at a slope of 1 in 8000 and carries a discharge of 800 litres/s when flowing half full. Take the value of Manning's N = 0.020.*

Solution. Given :

Slope of pipe, $i = \frac{1}{8000}$

Discharge, $Q = 800$ litres/s $= 0.8\ m^3/s$

Manning's, $N = 0.020$

Let the dia. of sewer pipe, $= D$

Depth of flow, $d = \frac{D}{2}$

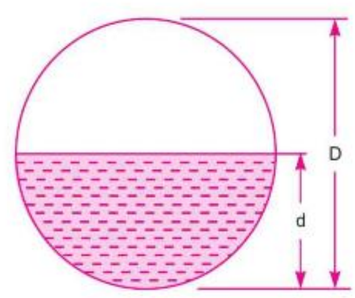


Fig. 16.8

∴ Area of flow, $A = \frac{\pi}{4}D^2 \times \frac{1}{2} = \frac{\pi D^2}{8}$

Wetted perimeter, $P = \frac{\pi D}{2}$

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi D^2}{8}}{\pi D/2} = \frac{D}{4}$

Using Manning's formula given by equation (16.8), $C = \frac{1}{N} m^{1/6}$

The discharge, Q through pipe is given by equation (16.6), as

$$Q = AC\sqrt{mi}$$

$$= \frac{\pi D^2}{8} \times \frac{1}{N} m^{1/6} \sqrt{mi}$$

or $$0.80 = \frac{\pi}{8} D^2 \times \frac{1}{.020} \times m^{1/6} \times m^{1/2} \times \sqrt{i}$$

$$= \frac{\pi}{8} D^2 \times \frac{1}{.020} m^{(1/6 + 1/2)} \times \sqrt{\frac{1}{8000}} = \frac{\pi}{8} D^2 \times \frac{1}{.020} \times m^{2/3} \times 0.01118$$

$$= 0.2195 \times D^2 \times \left(\frac{D}{4}\right)^{2/3} \qquad \left(\because \; m = \frac{D}{4}\right)$$

$$= \frac{.2195}{4^{2/3}} \times D^2 \times D^{2/3} = 0.0871\ D^{8/3}$$

or $$D^{8/3} = \frac{0.80}{.0871} = 9.1848$$

∴ $D = (9.1848)^{3/8} = (9.1848)^{0.375} =$ **2.296 m. Ans.**

Problem 16.13 *A rectangular channel of width, 4 m is having a bed slope of 1 in 1500. Find the maximum discharge through the channel. Take value of C = 50.*

Solution. Given :

Width of channel, $b = 4$ m

Bed slope, $i = \frac{1}{1500}$

Chezy's constant, $C = 50$

Discharge will be maximum, when the channel is most economical. The conditions for most economical rectangular channel are :

(*i*) $b = 2d$ or $d = \frac{b}{2} = \frac{4}{2} = 2.0$ m

(*ii*) $m = \frac{d}{2} = \frac{2}{2} = 1.0$ m

∴ Area of most economical rectangular channel, $A = b \times d = 4.0 \times 2.0 = 8\ m^2$

Using equation (16.5) for discharge as

$$Q = AC\sqrt{mi} = 8.0 \times 50 \times \sqrt{1.0 \times \frac{1}{1500}} = \mathbf{10.328\ m^3/s.\ Ans.}$$

Problem 16.14 *A rectangular channel carries water at the rate of 400 litres/s when bed slope is 1 in 2000. Find the most economical dimensions of the channel if C = 50.*

Solution. Given :

Discharge, $Q = 400 \text{ litres/s} = 0.4 \text{ m}^3\text{/s}$

Bed slope, $i = \frac{1}{2000}$

Chezy's constant, $C = 50$

For the rectangular channel to be most economical,

(*i*) Width, $b = 2d$

(*ii*) Hydraulic mean depth, $m = \frac{d}{2}$

∴ Area of flow, $A = b \times d = 2d \times d = 2d^2$

Using equation (16.5) for discharge,

$$Q = AC\sqrt{mi}$$

or

$$0.4 = 2d^2 \times 50 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}} = 2 \times 50 \times \sqrt{\frac{1}{2 \times 2000}}\ d^{5/2} = 1.581\ d^{5/2}$$

∴ $d^{5/2} = \frac{0.4}{1.581} = 0.253$

∴ $d = (.253)^{2/5} = \mathbf{0.577\ m.\ Ans.}$

$b = 2d = 2 \times .577 = \mathbf{1.154\ m.\ Ans.}$

Problem 16.15 *A rectangular channel 4 m wide has depth of water 1.5 m. The slope of the bed of the channel is 1 in 1000 and value of Chezy's constant C = 55. It is desired to increase the discharge to a maximum by changing the dimensions of the section for constant area of cross-section, slope of the bed and roughness of the channel. Find the new dimensions of the channel and increase in discharge.*

Solution. Given :

Width of channel, $b = 4.0$ m

Depth of flow, $d = 1.5$ m

∴ Area of flow, $A = b \times d = 4 \times 1.5 = 6.0 \text{ m}^2$

Slope of bed, $i = \frac{1}{1000}$

Chezy's constant, $C = 55$

Wetted perimeter, $P = d + b + d = 1.5 + 4 + 1.5 = 7.0$ m

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{4.0}{7.0} = 0.857$

The discharge, Q is given by $Q = AC\sqrt{mi} = 6.0 \times 55\sqrt{0.857 \times \frac{1}{1000}} = 9.66 \text{ m}^3\text{/s}$ ...(*i*)

For maximum discharge for a given area, slope of bed and roughness we proceed as :

Let b' = new width of channel

d' = new depth of flow

Then, Area, $A = b' \times d'$, where A = constant = 6.0 m^2

$\therefore$ $b' \times d' = 6.0$ ...(*ii*)

Also for maximum discharge $b' = 2d'$ ...(*iii*)

Substituting the value of b' in equation (*ii*), we have

$$2d' \times d' = 6.0 \text{ or } d'^2 = \frac{6.0}{2} = 3.0$$

$\therefore$ $d' = \sqrt{3} = 1.732$

Substituting the value of d' in (*iii*), we get

$$b' = 2 \times 1.732 = 3.464$$

$\therefore$ New dimensions of the channel are

Width, $b' =$ **3.464 m. Ans.**

Depth, $d' =$ **1.732 m. Ans.**

Wetted perimeter, $P' = d' + b' + d' = 1.732 + 3.464 + 1.732 = 6.928$

$\therefore$ Hydraulic mean depth, $m' = \frac{A}{P'} = \frac{6.0}{6.928} = 0.866$ m

(New hydraulic mean depth, m' corresponds to the condition of maximum discharge. And hence also equal to

$$\frac{d'}{2} = \frac{1.732}{2} = 0.866 \text{ m})$$

Max. discharge, Q', is given by $Q' = AC\sqrt{m'i} = 6.0 \times 55 \times \sqrt{0.866 \times \frac{1}{1000}} = 9.71 \text{ m}^3/\text{s}$ ...(*iv*)

$\therefore$ Increase in discharge $= Q' - Q = 9.71 - 9.66 =$ **0.05 m³/s. Ans.**

Problem 16.16 *A trapezoidal channel has side slopes of 1 horizontal to 2 vertical and the slope of the bed is 1 in 1500. The area of the section is 40 m². Find the dimensions of the section if it is most economical. Determine the discharge of the most economical section if C = 50.*

Solution. Given :

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{1}{2}$

Bed slope, $i = \frac{1}{1500}$

Area of section, $A = 40 \text{ m}^2$

Chezy's constant, $C = 50$

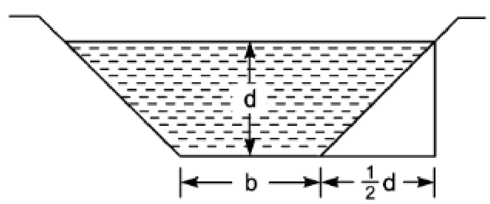


Fig. 16.12

For the most economical section, using equation (16.11)

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1} \quad \text{or} \quad \frac{b + 2 \times \frac{1}{2} \times d}{2} = d\sqrt{\left(\frac{1}{2}\right)^2 + 1}$$

or $$\frac{b + d}{2} = d\sqrt{\frac{1}{4} + 1} = 1.118\, d$$

or $$b = 2 \times 1.118d -- d = 1.236\, d \quad ...(i)$$

But area of trapezoidal section, $A = \frac{b + (b + 2nd)}{2} \times d = (b + nd)\, d$

$$= (1.236\, d + \tfrac{1}{2}\, d)\, d \qquad (\because \quad b = 1.236\, d \text{ and } n = \tfrac{1}{2})$$

$$= 1.736\, d^2$$

But $A = 40 \text{ m}^2$ (given)

$\therefore$ $40 = 1.736\, d^2$

$\therefore$ $d = \sqrt{\frac{40}{1.736}}$ = **4.80 m. Ans.**

Substituting the value of d in equation (i), we get

$b = 1.236 \times 4.80 =$ **5.933 m. Ans.**

Discharge for most economical section. Hydraulic mean depth for most economical section is

$$m = \frac{d}{2} = \frac{4.80}{2} = 2.40 \text{ m}$$

$\therefore$ Discharge $Q = AC\sqrt{mi} = 40 \times 50 \times \sqrt{2.40 \times \frac{1}{1500}}$

$= \mathbf{80\ m^3/s.\ Ans.}$

Problem 16.17 *A trapezoidal channel has side slopes of 3 horizontal to 4 vertical and slope of its bed is 1 in 2000. Determine the optimum dimensions of the channel, if it is to carry water at 0.5 m^3/s. Take Chezy's constant as 80.*

Solution. Given :

Side slopes $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{3}{4}$

Slope of bed, $i = \frac{1}{2000}$

Discharge, $Q = 0.5 \text{ m}^3/\text{s}$

Chezy's constant, $C = 80$

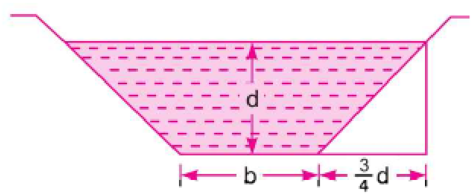


Fig. 16.13

For the most economical section, the condition is given by equation (16.11) as

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}, \text{ where } b = \text{width of section}, d = \text{depth of flow}$$

or $$\frac{b + 2 \times \frac{3}{4} d}{2} = d\sqrt{\left(\frac{3}{4}\right)^2 + 1} = \frac{5}{4} d \quad \text{or} \quad \frac{b + 1.5d}{2} = 1.25\, d$$

or $$b = 2 \times 1.25\, d - 1.5\, d = d \qquad \ldots(i)$$

For the discharge, Q, using equation (16.5) as

$$Q = AC\sqrt{mi} \qquad \ldots(ii)$$

But for most economical section, hydraulic mean depth $m = \frac{d}{2}$

Substituting the value of m and other known values in equation (*ii*)

$$0.50 = A \times 80 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}} \quad ...(iii)$$

But area of trapezoidal section is given as

$$A = (b + nd) \times d = (d + \tfrac{3}{4}d) \times d \quad (\because \text{ From } (i)\ b = d \text{ and } n = \tfrac{3}{4})$$

$$= \frac{7}{4}d^2 = 1.75\ d^2$$

Substituting the value of A in equation (*iii*), we get

$$0.50 = 1.75\ d^2 \times 80 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}} = 2.2135\ d^{5/2}$$

$$\therefore \quad d = \left(\frac{0.50}{2.2135}\right)^{2/5} = \mathbf{0.55\ m.\ Ans.}$$

From equation (*i*), $b = d =$ **0.55 m. Ans.**

$\therefore$ Optimum dimensions of the channel are width = depth = 0.55 m.

Problem 16.18 *A trapezoidal channel with side slopes of 1 to 1 has to be designed to convey 10 m³/s at a velocity of 2 m/s so that the amount of concrete lining for the bed and sides is the minimum. Calculate the area of lining required for one metre length of canal.*

Solution. Given :

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = 1$

Discharge $Q = 10\ m^3/s$

Velocity, $V = 2.0$ m/s

$\therefore$ Area of flow, $A = \frac{\text{Discharge}}{\text{Velocity}} = \frac{10.0}{2.0} = 5\ m^2 \quad ...(i)$

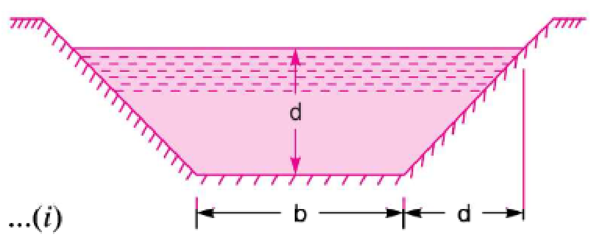


Fig. 16.14

Let b = Width of the channel

d = Depth of flow

For the amount of concrete lining for the bed and sides to be minimum the section should be most economical. But for the most economical trapezoidal section, the condition is from equation (16.11) as

Half of the top width = one of the sloping side

i.e., $$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

For $n = 1$, the condition becomes

$$\frac{b + 2 \times 1d}{2} = d\sqrt{1^2 + 1} = 1.414\ d$$

or $$b = 2 \times 1.414d - 2d = 0.828\ d \quad ...(ii)$$

But area, $$A = (b + nd)\ d = (0.828d + 1 \times d)\ d \quad (\because\ b = 0.828\ d,\ n = 1)$$

$$= 1.828\ d^2$$

Also from equation (*i*), $A = 5 \text{ m}^2$

Equating the two values of A, we get

$$5 = 1.828\, d^2 \quad \text{or} \quad d = \sqrt{\frac{5}{1.828}} = 1.6538 \simeq 1.654 \text{ m}$$

From equation (*ii*), $b = 0.828\, d = 0.828 \times 1.654 = 1.369 \text{ m}$

Area of lining required for one metre length of canal

= Wetted perimeter × length of canal

$= P \times 1$

where $P = b + 2d\sqrt{n^2+1} = 1.369 + 2 \times 1.654\sqrt{1^2+1} = 6.047 \text{ m}$

$\therefore$ Area of lining $= 6.047 \times 1 =$ **6.047 m². Ans.**

Problem 16.19 *A trapezoidal channel has side slopes 1 to 1. It is required to discharge 13.75 m³/s of water with a bed gradient of 1 in 1000. If unlined the value of Chezy's C is 44. If lined with concrete, its value is 60. The cost per m³ of excavation is four times the cost per m² of lining. The channel is to be the most efficient one. Find whether the lined canal or the unlined canal will be cheaper. What will be the dimensions of that economical canal ?*

Solution. Given :

Side slope, $n = \frac{1}{1} = 1$

Discharge, $Q = 13.75 \text{ m}^3/\text{s}$

Slope of bed, $i = \frac{1}{1000}$

For unlined, $C = 44$

For lined $C = 60$

Cost per m³ of excavation $= 4 \times$ cost per m² of lining

Let the cost per m² of lining $= x$

Then cost per m³ of excavation $= 4x$

As the channel is most efficient,

$\therefore$ Hydraulic mean depth, $m = \frac{d}{2}$, where d = depth of channel

Let b = width of channel

Also for the most efficient trapezoidal channel, from equation (16.11), we have

Half of top width = length of sloping side

or
$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

or
$$\frac{b + 2 \times 1 \times d}{2} = d\sqrt{1^2+1} = \sqrt{2}d$$

$\therefore$ $b = 2 \times \sqrt{2}d - 2d = 0.828\, d$ ...(*i*)

Area, $A = (b + nd) \times d = (0.828\, d + 1 \times d) \times d$

$= 1.828\, d^2$ ...(*ii*)

1. **For unlined channel**

Value of $C = 44$

The discharge, Q is given by, $Q = A \times V = A \times C\sqrt{mi}$

or $$13.75 = 1.828\ d^2 \times 44 \times \sqrt{\frac{d}{2} \times \frac{1}{1000}} \qquad \left(\because \quad A = 1.828\ d^2,\ m = \frac{d}{2}\right)$$

$$= \frac{1.828 \times 44}{\sqrt{2000}} \times d^{5/2}$$

$$d^{5/2} = \frac{13.75 \times \sqrt{2000}}{1.828 \times 44} = 7.6452$$

$\therefore$ $$d = (7.6452)^{2/5} = 2.256 \text{ m}$$

Substituting this value in equation (i), we get

$$b = 0.828 \times 2.256 = 1.868 \text{ m.}$$

Now cost of excavation per running metre length of unlined channel

$= $ Volume of channel $\times$ cost per m^3 of excavation

$= $ (Area of channel $\times$ 1) $\times 4x = [(b + nd) \times d \times 1] \times 4x$

$= (1.868 + 1 \times 2.256) \times 2.256 \times 1 \times 4x = 37.215\ x$ ...(iii)

2. **For lined channels**

Value of $C = 60$

The discharge is given by the equation, $Q = A \times C \times \sqrt{mi}$

Substituting the value of A from equation (ii) and value of $m = \frac{d}{2}$, we get

$$13.75 = 1.828\ d^2 \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{1000}} \qquad (\because \quad Q = 13.75)$$

$$= 1.828 \times 60 \times \frac{1}{\sqrt{2000}} \times d^{5/2}$$

$\therefore$ $$d^{5/2} = \frac{13.75 \times \sqrt{2000}}{1.828 \times 60} = 5.606$$

$\therefore$ $$d = (5.606)^{2/5} = 1.992 \text{ m}$$

Substituting this value in equation (i), we get

$$b = 0.828 \times 1.992 = 1.649 \text{ m}$$

In case of lined channel, the cost of lining as well as cost of excavation is to be considered.

Now cost of excavation $= $ (Volume of channel) $\times$ cost per m^3 of excavation

$= (b + nd) \times d \times 1 \times 4x$

$= (1.649 + 1 \times 1.992) \times 1.992 \times 1 \times 4x = 29.01\ x$

Cost of lining $= $ Area of lining $\times$ cost per m^2 of lining

$= $ (Perimeter of lining $\times$ 1) $\times x$

$$= (b + 2d\sqrt{1+n^2}) \times 1 \times x = (1.649 + 2 \times 1.992\sqrt{1+1^2}) \times 1 \times x$$

$$= (1.649 + 2 \times 1.992 \times \sqrt{2})\, x = 7.283\, x$$

$\therefore$ Total cost $= 29.01x + 7.283x = 36.293x$

The total cost of lined channel is $36.293x$ whereas the total cost of unlined channel is $37.215x$. Hence lined channel will be cheaper. The dimensions are $b = 1.649$ m and $d = 1.992$ m. **Ans.**

Problem 16.20 *An open channel of most economical section, having the form of a half hexagon with horizontal bottom is required to give a maximum discharge of 20.2 m³/s of water. The slope of the channel bottom is 1 in 2500. Taking Chezy's constant, C = 60 in Chezy's equation, determine the dimensions of the cross-section.*

Solution. Given :

Maximum discharge, $Q = 20.2 \text{ m}^3/\text{s}$

Bed slope, $i = \dfrac{1}{2500}$

Chezy's constant, $C = 60$

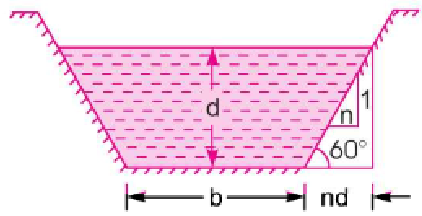


Fig. 16.15

Channel is the form of a half hexagon as shown in Fig. 16.15. This means that the angle made by the sloping side with horizontal will be 60°.

$$\therefore \quad \tan\theta = \tan 60° = \sqrt{3} = \frac{1}{n}$$

$$\therefore \quad n = \frac{1}{\sqrt{3}}$$

Let b = width of the channel, d = depth of the flow.

As the channel given is of most economical section, hence the condition given by equations (16.11) and (16.12) should be satisfied *i.e.*,

Half of the top width = one of the sloping side

And hydraulic mean depth = half of depth of flow

From equation (16.11), $\dfrac{b+2nd}{2} = d\sqrt{n^2+1}$

For $n = \dfrac{1}{\sqrt{3}}$, $\quad \dfrac{b + 2 \times \dfrac{1d}{\sqrt{3}}}{2} = d\sqrt{\left(\dfrac{1}{\sqrt{3}}\right)^2 + 1} = \dfrac{2d}{\sqrt{3}}$

or $\quad \dfrac{\sqrt{3}b + 2d}{2\sqrt{3}} = \dfrac{2d}{\sqrt{3}} \quad$ or $\quad \dfrac{\sqrt{3}b + 2d}{2} = 2d$

$$\therefore \quad b = \frac{2 \times 2d - 2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}} \qquad \ldots(i)$$

Area of flow, $\quad A = (b + nd)\, d = \left(\dfrac{2}{\sqrt{3}}d + \dfrac{d}{\sqrt{3}}\right)d \qquad \left(\because \; n = \dfrac{1}{\sqrt{3}},\, b = \dfrac{2d}{\sqrt{3}}\right)$

$$= \frac{3}{\sqrt{3}} d^2 = \sqrt{3} d^2$$

From equation (16.12) $\quad m = \frac{d}{2}$

Using equation (16.5) for discharge Q as

$$Q = AC\sqrt{mi} \quad \text{or} \quad 20.2 = \sqrt{3}\ d^2 \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{2500}} = 1.4694\ d^{5/2}$$

$\therefore$ $$d^{5/2} = \frac{20.2}{1.4696} = 13.745$$

$\therefore$ $$d = (13.745)^{2/5} = \mathbf{2.852\ m.\ Ans.}$$

Substituting this value in equation (*i*), we get

$$b = \frac{2d}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times 2.852 = \mathbf{3.293\ m.\ Ans.}$$

Problem 16.21 *A trapezoidal channel to carry 142 m³/minute of water is designed to have a minimum cross-section. Find the bottom width and depth if the bed slope is 1 in 1200, the side slopes at 45° and Chezy's co-efficient = 55.*

Solution. Given : Discharge, $Q = 142\ \text{m}^3/\text{min.} = \frac{142}{60} = 2.367\ \text{m}^3/\text{s}$

Bed slope, $\quad i = 1 \text{ in } 1200 = \frac{1}{1200}$

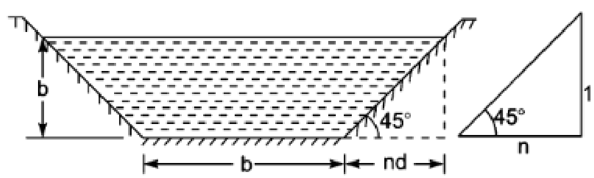


Fig. 16.16

Side slope, $\quad \theta = 45°$

$\therefore$ $\quad \tan\theta = \frac{1}{n} \quad \text{or} \quad \tan 45° = \frac{1}{n}$

$\therefore$ $\quad 1 = \frac{1}{n} \quad \text{or} \quad n = 1$

Chezy's constant, $\quad C = 55$

Let $\quad b$ = Width of the channel, d = Depth of the flow.

As the channel is to be designed for a minimum cross-section (*i.e.*, channel is of most economical section), the conditions given by equations (16.11) and (16.12) should be satisfied *i.e.*,

(*i*) Half of top with = Length of sloping side

(*ii*) Hydraulic mean depth = Half of depth of flow

From equation (16.11), $\frac{b+2nd}{2} = d\sqrt{n^2+1}$

or $$\frac{b+2\times1\times d}{2} = d\sqrt{1^2+1} \qquad (\because\ n = 1)$$

or $$b + 2d = 2\sqrt{2}d = 2\times1.414\,d = 2.828\,d$$

$$\therefore \quad b = 2.828\,d - 2d = 0.828\,d \qquad ...(i)$$

Now using equation (16.5) for discharge Q, we get

$$Q = A\,.\,C\,.\,\sqrt{mi}$$

or $$2.367 = (b+nd)\,d\times55\sqrt{\frac{d}{2}\times\frac{1}{1200}} \qquad \left(\because\ A = (b+nd)\times d \text{ and } m = \frac{d}{2}\right)$$

$$= (0.828d\times1\times d)\,d\times55\sqrt{\frac{d}{2400}} \qquad (\because\ b = 0.828d)$$

$$= (1.828d)\times d\times55\sqrt{\frac{d}{2400}} = 2.052\,d^{5/2}$$

$$\therefore \quad d = \left(\frac{2.367}{2.052}\right)^{2/5} = 1.058 \simeq \mathbf{1.06\ m.\ Ans.}$$

Substituting this value in equation (*i*), we get

$$b = 0.828\times1.06 = \mathbf{0.877\ m.\ Ans.}$$

Problem 16.22 *A trapezoidal channel with side slopes of 3 horizontal to 2 vertical has to be designed to convey 10 m^3/s at a velocity of 1.5 m/s, so that the amount of concrete lining for the bed and sides is minimum. Find*

(i) the wetted perimeter, and

(ii) slope of the bed if Manning's N = 0.014 in the formula $C = \frac{1}{N}\times m^{1/6}$

Solution. (*i*) Given :

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{3}{2} = 1.5$

Discharge, $Q = 10\ m^3/s$

Velocity, $V = 1.5$ m/s

Manning's constant, $N = .014$

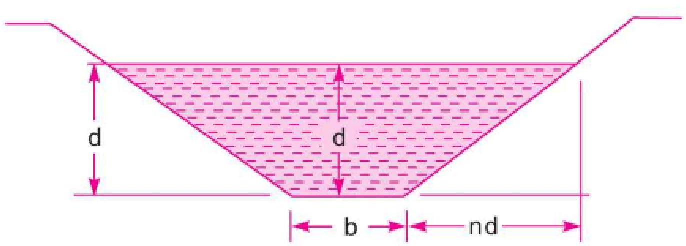


Fig. 16.17

Let b = width of the channel, d = depth of the flow.

The amount of concrete lining for the bed and sides will be minimum, when the section is most economical. For most economical trapezoidal section, the condition is given by equation (16.11) as

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

For $n = 1.5$, $\frac{b + 2 \times 1.5 \times d}{2} = d\sqrt{1.5^2 + 1} = \sqrt{3.25}\, d = 1.8\ d$ or $\frac{b + 3d}{2} = 1.8\ d$

$\therefore$ $b = 2 \times 1.8 - 3d = 0.6\ d$ ...(i)

But area of trapezoidal section, $A = (b + nd)d = (0.6d + 1.5d)d$ ($\because$ $b = 0.6d$, $n = 1.5$)

$= 2.1\ d^2$

Also area, $A = \frac{\text{Discharge}}{\text{Velocity}} = \frac{Q}{V} = \frac{10.0}{1.5} = 6.67\ \text{m}^2$

Equating the two values of A, we have $2.1\ d^2 = 6.67$

$\therefore$ $d = \sqrt{\frac{6.67}{2.1}} = 1.78$ m

From equation (i), $b = 0.6d = 0.6 \times 1.78 = 1.068 \simeq 1.07$ m

Hence wetted perimeter, $P = b + 2d\sqrt{n^2 + 1} = 1.07 + 2 \times 1.78\sqrt{1.5^2 + 1}$ = **7.48 m. Ans.**

(ii) Slope of the bed when $N = 0.014$ in the formula, $C = \frac{1}{N}\ m^{1/6}$

For the most economical trapezoidal section, hydraulic mean depth m, is given by equation (16.12) as

$$m = \frac{d}{2} = \frac{1.78}{2} = 0.89\ \text{m}$$

$$C = \frac{1}{0.014} \times (.89)^{1/6} = 66.09$$

Using equation (16.5), $Q = AC\sqrt{mi}$

or $10.0 = 6.67 \times 66.09\sqrt{0.89} \times i = 415.86\sqrt{i}$

$\therefore$ $i = \left(\frac{10}{415.86}\right)^2 = \mathbf{\frac{1}{1729.4}}$. **Ans.**

Hence slope of the bed is 1 in 1729.4.

Problem 16.23 *A power canal of trapezoidal section has to be excavated through hard clay at the least cost. Determine the dimensions of the channel given, discharge equal to 14 m^3/s, bed slope 1 : 2500 and Manning's N = 0.020.*

Solution. Given :

Discharge, $Q = 14 \text{ m}^3/\text{s}$

Bed slope, $i = \dfrac{1}{2500}$

Manning's, $N = 0.020$

For excavation of the canal at the least cost, the trapezoidal section should be most economical. Here side slope (*i.e.*, value of *n*) is not given. Hence the best side slope for most economical trapezoidal section (*i.e.*, the value of *n*) is given by equation (16.14) as $n = \dfrac{1}{\sqrt{3}}$

Let b = width of channel, d = depth of flow

For most economical section,

Half of top width = length of one of sloping side

or
$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

For $n = \dfrac{1}{\sqrt{3}}$,
$$\frac{b + 2 \times \frac{1}{\sqrt{3}} d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} = \frac{2d}{\sqrt{3}}$$

or
$$b = \frac{2 \times 2d}{\sqrt{3}} - \frac{2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}} \qquad \ldots(i)$$

Area of trapezoidal section,
$$A = (b + nd) \times d = \left(\frac{2d}{\sqrt{3}} + \frac{1}{\sqrt{3}} d\right) \times d \qquad \left(\because \; b = \frac{2d}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}\right)$$
$$= \sqrt{3}d^2$$

Hydraulic mean depth for most economical section, $m = \dfrac{d}{2}$

Now discharge, Q is given by $Q = AC\sqrt{mi}$, where $C = \dfrac{1}{N} m^{1/6}$

$\therefore$
$$Q = \sqrt{3}d^2 \times \frac{1}{N} m^{1/6} \sqrt{m \times \frac{1}{2500}}$$
$$= \sqrt{3}d^2 \times \frac{1}{0.020} \times m^{1/6 + 1/2} \times \sqrt{\frac{1}{2500}} = 1.732\, d^2 \times m^{2/3}$$

or $$14.0 = 1.732\, d^2 \times \left(\frac{d}{2}\right)^{2/3} = \frac{1.732}{2^{2/3}} d^{8/3} = 1.09\, d^{8/3}$$

$\therefore$ $$d^{8/3} = \frac{14.0}{1.09} = 12.844$$

$\therefore$ $$d = (12.844)^{3/8} = (12.844)^{0.375} = \mathbf{2.605\ m.\ Ans.}$$

From equation (*i*), $$b = \frac{2d}{\sqrt{3}} = \frac{2 \times 2.605}{1.732} = \mathbf{3.008\ m.\ Ans.}$$

Problem 16.24 *For a trapezoidal channel with bottom width 40 m and side slopes 2H : 1 V, Manning's N is 0.015 and bottom slope is 0.0002. If it carries 60 m³/s discharge, determine the normal depth.*

Solution. Given :

Bottom width, $b = 40$ m

Side slopes 2 horizontal to 1 vertical *i.e.*, $n = 2$

$\therefore$ Manning's constant, $N = 0.015$

Bed slope, $i = 0.0002$

Discharge, $Q = 60\ m^3/s$

Let d = Normal depth.

Now $$A = (b + nd) \times d = (40 + 2d) \times d$$

$$P = b + 2d\sqrt{1+n^2} = 40 + 2d\sqrt{1+2^2} = 40 + 2 \times \sqrt{5}\, d = 40 + 4.472\, d$$

$\therefore$ $$m = \frac{A}{P} = \frac{(40+2d)\times d}{40+4.472d}$$

The discharge is given by, $$Q = \text{Area} \times \text{Velocity}$$

$$= A \times \frac{1}{N} m^{2/3}\, i^{1/2} = \frac{A}{N} \times m^{2/3} \times i^{1/2}$$

$$60 = \frac{(40+2d)\times d}{0.015} \times \left[\frac{(40+2d)\times d}{40+4.472d}\right]^{2/3} \times 0.0002^{1/2}$$

$$= \frac{[(40+2d)\times d]^{5/3}}{0.015 \times (40+4.472d)^{2/3}} \times 0.01414$$

$$\therefore\ \frac{60 \times 0.015 \times (40+4.472d)^{2/3}}{0.01414} = [(40 + 2d) \times d]^{5/3}$$

$$63.65(40 + 4.472\ d)^{2/3} = (40d + 2d^2)^{5/3}$$

$$(40d + 2d^2)^{5/3} - 63.65\ (40 + 4.472d)^{2/3} = 0 \qquad \ldots(i)$$

The above equation will be solved by Hit and Trial method.

(*i*) Assume d = 1 m, then L.H.S of equation (*i*) will as

$$\text{L.H.S.} = (40 + 2)^{5/3} - 63.65\,(40 + 4.472)^{2/3}$$
$$= 42^{5/3} - 63.65 \times 44.472^{2/3} = 513.838 - 808.4 = -294.56$$

(*ii*) Assume d = 2 m, then L.H.S. of equation (*i*) will be as

$$\text{L.H.S.} = (40 \times 2 + 2 \times 2^2)^{5/3} - 63.65(40 + 4.47 \times 2)^{2/3}$$
$$= 88^{5/3} - 63.65 \times 48.944^{2/3} = 1767.2 - 862.77 = 904.43$$

where d = 1 m, L.H.S. is – ve. But when d = 2 m, L.H.S. is +ve. Hence value of d lies between 1 and 2.

(*iii*) Assume d = 1.3 m, then L.H.S. of equation (*i*) will be as

$$\text{L.H.S.} = (40 \times 1.3 + 2 \times 1.3^2)^{5/3} - 63.65(40 + 4.472 \times 1.3)^{2/3}$$
$$= 55.38^{5/3} - 63.65 \times 45.8136^{2/3} = 815.45 - 825.4 = -9.95$$

(*iv*) Assume d = 1.31 m, then L.H.S. of equation (*i*) will be

$$\text{L.H.S.} = (40 \times 1.31 + 2 \times 1.31^2)^{5/3} - 63.65(40 + 4.472 \times 1.31)^{2/3}$$
$$= 55.8322^{5/3} - 63.65 \times 45.8583^{2/3} = 826.6 - 825.9 = 0.7$$

The value of L.H.S. = 0.7 is negligible in comparison to the value of 904.43.

∴ Value of d = **1.31 m. Ans.**

Problem 16.25 *Find the discharge through a circular pipe of diameter 3.0 m, if the depth of water in the pipe is 1.0 m and the pipe is laid at a slope of 1 in 1000. Take the value of Chezy's constant as 70.*

Solution. Given :

Dia. of pipe, $D = 3.0$

∴ Radius, $R = \dfrac{D}{2} = \dfrac{3.0}{2} = 1.50$ m

Depth of water in pipe, $d = 1.0$ m

Bed slope, $i = \dfrac{1}{1000}$

Chezy's constant, $C = 70$

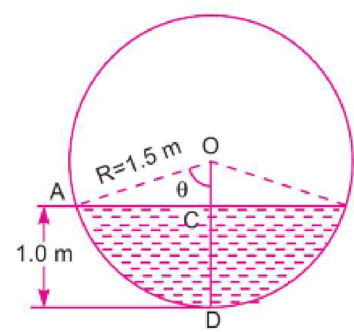


Fig. 16.20

From Fig. 16.20, we have $OC = OD - CD = R - 1.0$

$= 1.5 - 1.0 = 0.5$ m

$AO = R = 1.5$ m

Also $$\cos\theta = \frac{OC}{AO} = \frac{0.5}{1.5} = \frac{1}{3}$$

∴ $$\theta = 70.53° = 70.53 \times \frac{\pi}{180} = 1.23 \text{ radians} \qquad (\because 180° = \pi \text{ radians})$$

Wetted perimeter is given by equation (16.16) as

$$P = 2R\theta = 2 \times 1.5 \times 1.23 \qquad (\theta \text{ should be in radians})$$
$$= 3.69 \text{ m}$$

Wetted area is given by equation (16.17) as

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 1.5^2\left(1.23 - \frac{\sin(2\times 70.53°)}{2}\right)$$

$$= 2.25\left[1.23 - \frac{\sin(141.08°)}{2}\right] = 2.25\left[1.23 - \frac{\sin(180°-141.08°)}{2}\right]$$

$$= 2.25\left[1.23 - \frac{\sin 38.94°}{2}\right] = 2.06\ m^2$$

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{2.06}{3.69} = .5582$

The discharge is given by, $Q = AC\sqrt{mi} = 2.06 \times 70 \times \sqrt{0.5582 \times \frac{1}{1000}}$ **= 3.407 m²/s. Ans.**

Problem 16.26 *If in the problem 16.25, the depth of water in the pipe is 2.5 m, find the rate of flow through the pipe.*

Solution. Given :

Dia. of pipe $= 3.0$ m

∴ Radius, $R = 1.5$ m

Depth of water, $d = 2.5$ m

$$i = \frac{1}{1000} \text{ and } C = 70$$

From Fig. 16.21, $OC = CD - OD = 2.5 - R = 2.5 - 1.5 = 1.0$ m

$OA = R = 1.5$ m

From ΔAOC, $\cos\alpha = \frac{OC}{OA} = \frac{1.0}{1.5} = 0.667$

∴ $\alpha = 48.16°$

$$\theta = 180° - \alpha = 180° - 48.16° = 131.84°$$

$$= 131.84 \times \frac{\pi}{180} = 2.30 \text{ radians}$$

Now wetted perimeter is given by equation (16.16) as

$$P = 2R\theta = 2 \times 1.5 \times 2.30 = 6.90\ m$$

And wetted area is given by equation (16.17) as

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 1.5^2\left(2.30 - \frac{\sin(2\times 131.84°)}{2}\right)$$

$$= 2.25\left(2.30 - \frac{\sin 263.68°}{2}\right)$$

$$= 2.25\left[2.30 - \frac{\sin(180° + 83.68°)}{2}\right]$$

$$= 2.25\left[2.30 - \frac{(-\sin 83.58°)}{2}\right]$$

$$= 2.25\left[2.30 + \frac{\sin 83.68°}{2}\right] = 6.293 \text{ m}^2$$

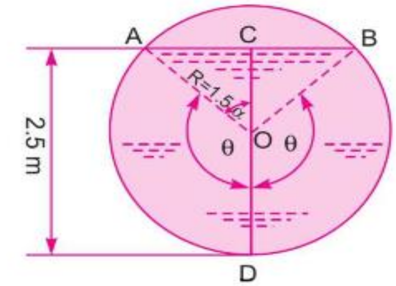


Fig. 16.21

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{6.293}{6.90} = 0.912$ m

Discharge, Q is given by, $Q = AC\sqrt{mi} = 6.293 \times 70 \times \sqrt{0.912 \times \frac{1}{1000}}$ = **13.303 m³/s. Ans.**

Problem 16.27 *Calculate the quantity of water that will be discharged at a uniform depth of 0.9 m in a 1.2 m diameter pipe which is laid at a slope 1 in 1000. Assume Chezy's C = 58.*

Solution. Given :

Dia. of pipe = 1.2 m

∴ Radius, $R = \frac{1.2}{2} = 0.6$ m

Depth of water, $d = 0.9$ m

Slope, $i = \frac{1}{1000}$

Chezy's, $C = 58$

From Fig. 16.22, we have $OC = CD - OD$

$= 0.9 - R = 0.9 - 0.6 = 0.3$ m

$OA = R = 0.6$ m

Now in triangle AOC,

$$\cos\alpha = \frac{OC}{OA} = \frac{0.3}{0.6} = \frac{1}{2}$$

$$\therefore \quad \alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

$$\therefore \quad \theta = \text{Angle } DOA = 180° - \alpha$$

$$= 180° - 60° = 120° = 120 \times \frac{\pi}{180} \text{ radians}$$

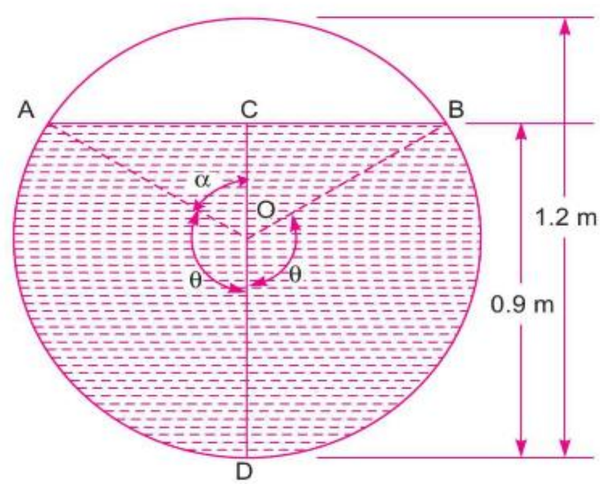


Fig. 16.22

$$= 0.667\ \pi \text{ radians}$$

Now wetted perimeter is given by equation (16.16) as

$$P = 2R\theta = 2 \times 0.6 \times 0.667\ \pi = 2.526 \text{ m}$$

And area of flow is given by equation (16.17) as

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$= 0.6^2\left[0.667\pi - \frac{\sin(2 \times 120°)}{2}\right] = 0.36\left[0.667\pi - \frac{\sin 240°}{2}\right]$$

$$= 0.36\left[0.667\pi - \frac{(-0.866)}{2}\right] = 0.36\ [0.667\ \pi + 0.433] = 0.913 \text{ m}^2$$

Now discharge is given by, $Q = A \times V = A \times C\sqrt{mi} = 0.913 \times 58\sqrt{\frac{A}{P} \times \frac{1}{1000}}$ $\left(\because\ m = \frac{A}{P}\right)$

$$= 0.913 \times 58\sqrt{\frac{0.913}{2.526} \times \frac{1}{1000}} = \mathbf{1.007\ m^3/s.\ Ans.}$$

Problem 16.28 *Water is flowing through a circular channel at the rate of 400 litres/s, when the channel is having a bed slope of 1 in 9000. The depth of water in the channel is 8.0 times the diameter. Find the diameter of the circular channel if the value of Manning's N = 0.015.*

Solution. Given :

Discharge, $Q = 400 \text{ litres/s} = 0.4 \text{ m}^3\text{/s}$

Bed slope, $i = \frac{1}{9000}$

Manning's, $N = 0.015$

Let the diameter of channel $= D$

Then depth of flow, $d = 0.8\ D$

From Fig. 16.23, we have

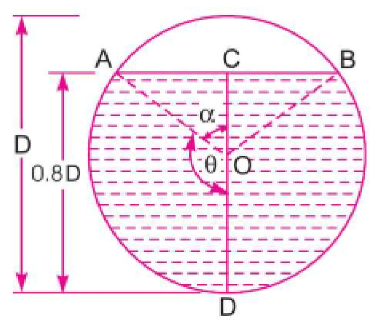


Fig. 16.23

$$OC = CD - OD = 0.8\ D - \frac{D}{2}$$

$$= (0.8 - 0.5)\ D = 0.3\ D$$

And $AO = R = \frac{D}{2} = 0.5\ D$

$$\therefore \quad \cos\alpha = \frac{OC}{AO} = \frac{0.3\ D}{0.5\ D} = 0.6$$

$$\therefore \quad \alpha = 53.13°$$

And $\theta = 180° - 53.13 = 126.87° = 126.87 \times \frac{\pi}{180} = 2.214$ radians.

From equation (16.16), wetted perimeter,

$$P = 2R\theta = 2 \times \frac{D}{2} \times 2.214 = 2.214\ D \text{ m.}$$

From equation (16.17), wetted area,

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = \left(\frac{D}{2}\right)^2 \left[2.214 - \frac{\sin(2 \times 126.87°)}{2}\right]$$

$$= \frac{D^2}{4}\left[2.214 - \frac{\sin 253.74°}{2}\right] = \frac{D^2}{4}\left[2.214 - \frac{\sin(180°+73.74°)}{2}\right]$$

$$= \frac{D^2}{4}\left[2.214 - \left(\frac{-\sin 73.74°}{2}\right)\right] = \frac{D^2}{4}\left[2.214 + \frac{\sin 73.74°}{2}\right]$$

$$= \frac{D^2}{4}\ [2.214 + .48] = 0.6735\ D^2$$

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{0.6735\ D^2}{2.214\ D} = 0.3042\ D$

Discharge by Manning's formula is given by,

$$Q = \frac{1}{N} \times A \times m^{2/3} \times i^{1/2}$$

or

$$0.4 = \frac{1}{.015} \times 0.6735\ D^2 \times (.3042\ D)^{2/3} \times \left(\frac{1}{9000}\right)^{1/2}$$

$$= \frac{0.6735}{0.015}\ D^2 \times 34521 \times D^{2/3} \times 0.0105 = 0.213\ D^{8/3}$$

$\therefore$ $$D^{8/3} = \frac{0.40}{0.213} = 1.8779$$

$\therefore$ $$D = (1.8779)^{3/8} = (1.8779)^{0.375} = \mathbf{1.266\ m.\ Ans.}$$

Problem 16.29 *A sewer pipe is to be laid at a slope of 1 in 8100 to carry a maximum discharge of 600 litres/s, when the depth of water is 75% of the vertical diameter. Find the diameter of this pipe if the value of Manning's N is 0.025.*

Solution. Given :

Discharge, $Q = 600$ litres/s $= 0.6\ m^3/s$

Bed slope, $i = \frac{1}{8100}$

Manning's, $N = 0.025$

Depth of water = 75% of dia. of pipe = 0.75 dia. of pipe

Let d = depth of water, D = Dia. of pipe

Then $d = 0.75\ D$

From Fig. 16.23 (*a*), we have $OC = CD - OD = 0.75\ D - 0.5\ D = 0.25\ D$

$$AO = R = 0.5\ D$$

In triangle AOC, $\cos\alpha = \dfrac{OC}{AO} = \dfrac{0.25\ D}{0.5\ D} = 0.5$

$\therefore$ $\alpha = \cos^{-1} 0.5 = 60°$

And $\theta = 180° - \alpha = 180° - 60° = 120°$

$$= 120 \times \frac{\pi}{180} \text{ radians} = 2.0946 \text{ radians.}$$

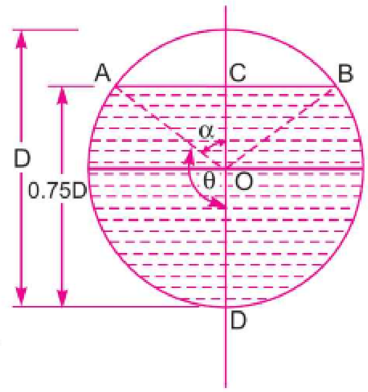


Fig. 16.23 (*a*)

From equation (16.16), wetted perimeter

$$P = 2R\theta = 2 \times 0.5\ D \times 2.0496 \qquad (\because\ R = 0.5\ D)$$

$$= 2.0496\ D$$

And from equation (16.17), the area of flow,

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$= (0.5\ D)^2 \left[2.0946 - \frac{\sin(2 \times 120°)}{2}\right]$$

$$= 0.25\ D^2 \left[2.0946 - \left(\frac{-.866}{2}\right)\right] = 0.25\ D^2\ [2.0946 + 0.433]$$

$$= 0.6319\ D^2$$

$\therefore$ $m = \dfrac{A}{P} = \dfrac{0.6319\ D^2}{2.0496\ D} = 0.308\ D$

Discharge by Manning's formula is given by

$$Q = \frac{1}{N} \times A \times m^{2/3} \times i^{1/2}$$

or $0.6 = \dfrac{1}{0.025} \times 0.6319\ D^2 \times (0.308\ D)^{2/3} \times \left(\dfrac{1}{8100}\right)^{1/2} = 0.128 \times D^{8/3}$

$\therefore$ $D^{8/3} = \dfrac{0.6}{0.128} = 4.6875$

$\therefore$ $D = (4.6875)^{3/8} =$ **1.785 m. Ans.**